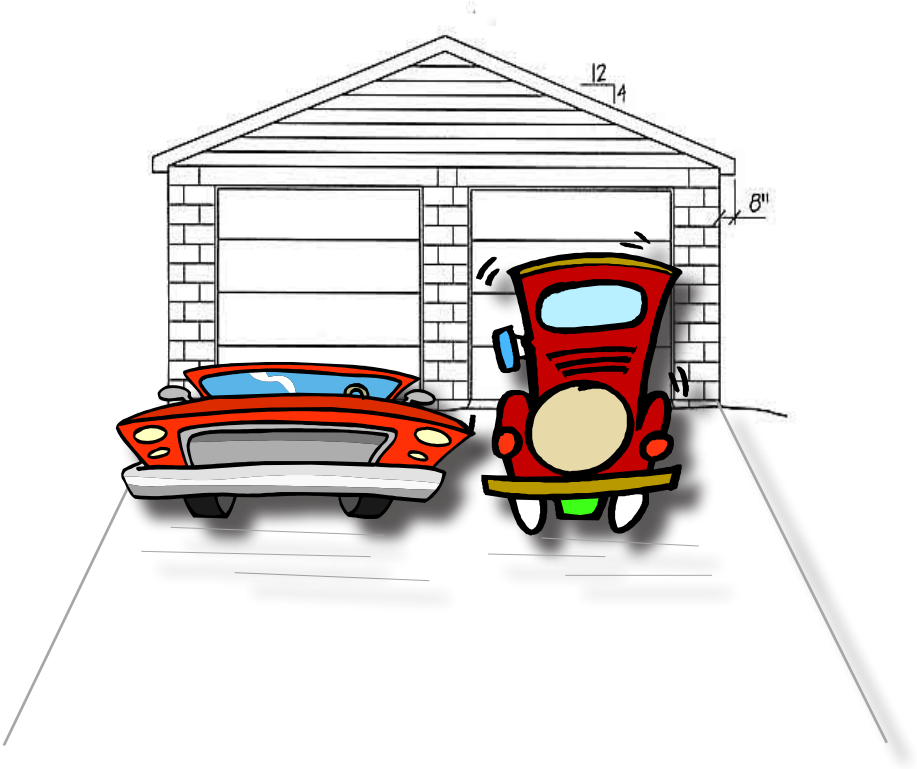
Garage Patterns



Exploring Patterns and Equations

Outcomes

- Use manipulatives to explore patterns.
- Connect pattern explorations to the NCTM Standards and algebraic thinking.
- Become aware of the changes that have taken place in mathematics education.

Overview

This module uses the context of building garages with unifix cubes to present algebraic concepts. Participants build patterns with blocks, record the patterns in tables, write rules for the patterns, and then graph them. They use the tables, rules and graphs to find information.

The participants start by finding sequences in **Searching for Patterns**. The last sequence is left open so that participants can create their own. This can lead to a fun discussion as participants try to guess each other's sequences. These sequences lead into the idea that pattern finding is an important skill in mathematics.

Next, participants build model garages out of unifix cubes. As participants build their garages, they fill in a table. They look for patterns and discuss what they find. Then they write a rule about how to find the number of blocks needed for making garages. Volunteers share their rules. All rules are written on a chart and tested to see if they work for several cases.

Participants explore another way for recording patterns: graphing. The facilitator models graphing while participants complete one on their own sheet. The participants are asked about the rate of change, but are not asked about slope. Since this is a short session to expose parents to mathematics today, the concept rather than the vocabulary is being emphasized.

Afterwards, all four methods (the model, table, rule and graph) are used to find the number of blocks needed for other structures.

The session's activities are then connected to the NCTM Principles and Standards for Teaching Mathematics. Participants are given three of the standards: the **Algebra Standard,** the **Connections Principle,** and the **Communication Principle**. The groups think about how today's approach to mathematics is different from the approach when they were in school. They examine the standards to see which ones were used in the session and find an example of when it was used.

Several patterning activities are included for parent and students to do at home.

Mathematics Background

The mathematical focus of this module is linear equations, and representing them using a model, a table, a rule, and a graph.

An understanding of patterns is basic to the understanding of algebra. In the NCTM Principles and Standards for School Mathematics, the progression through the grades is discussed. Early experiences with classifying and ordering objects should be the first steps in looking at patterns. It is suggested that teachers help children notice patterns and make predictions. If colored blocks alternate: red, blue, red, blue; what color do you predict the 12th block is? In the lower grades, students can describe patterns like 2, 4, 6,. . . by focusing on how a term is obtained from the previous number (by adding 2). This is the beginning of recursive thinking (a discussion of recursive and closed formulas is on the next page). In the middle school, students should focus on understanding linear relationships. They should also understand the relationships among tables, graphs, and symbols and judge the advantages and disadvantages of each way of representing relationships. As they progress through high school, students should develop an understanding of many types of functions. These NCTM guidelines have been used in developing the activities of Garage Patterns.

Garage Patterns introduces many algebraic concepts without using the vocabulary of algebra. Research supports the idea of developing concepts before a vocabulary word is introduced. However, the vocabulary is important, and in Math for Parents (Thinking in Patterns) the proper vocabulary is introduced. The Math Awareness Workshops are designed as introductions to teaching in alignment with the NCTM Principles and Standards for School Mathematics.

Below are some topics that might come up in the discussion with participants.

Algebraic Notation: The Language of Algebra
Participants will usually generate a statement of B = 5g for the first garage pattern. This is perfectly acceptable. Some participants may use the algebraic notation y = 5x. They would represent the number of cubes needed, and the x would represent the number of garages that could be built. If a participant uses this notation, it will take some time to process it. Have the participant clarify what the y and x represent and illustrate how this works. Then reaffirm that the statement # of blocks = 5 times # of garages (B = 5g) describes the same relationship.

In the second problem, Connected Garages, 5 blocks are needed to build the first garage, however each successive garage needs only an additional 3 blocks. This relationship is written in algebraic terms as B = 3g + 2, or B = 2 + 3g.

Linear Functions
The fact that these two graphs are both linear might be discussed. A linear function is one whose graph is a straight line. In the first pattern, the blocks consistently increase by 5, and in the second pattern, the blocks consistently increase by 3. Thus both patterns form straight lines when graphed.

Mathematics Background (continued)

Recursive and Closed Formulas
As participants come up with rules, some will be closed and some recursive. A recursive rule or formula, is one that uses the previous entry to find the new one. For example, in the first garage pattern, the blocks are increasing by 5 each time a new garage is built. If a participant comes up with the rule that they add 5 to the last total, it is an example of a recursive formula. In order to find the new total, it is necessary to know the last total. Recursive formulas are very useful for computer spreadsheets.

In a closed formula, the formula can be applied to any entry without knowing what has come before. If a participant says that they find the number of blocks by multiplying the number of garages by 5, he/she has stated a closed formula because it can be applied to any number of garages without knowing how many blocks were needed to build the number before. If there are 5 garages, there must be 5 X 5 blocks needed, or 25. To move from recursive to closed, record the patterns on the tables for the participants to see the relationship between adding 5's (repeated addition) and multiplying by 5. A note to the facilitator illustrates this process.

Independent and Dependent Variables
Independent and dependent variables might come up in the conversation in this module. The independent variable is represented on the x-axis, or the horizontal axis, while the dependent variable is represented on the y-axis, or the vertical one on the side. The independent variable is the one that causes the dependent variable to change. For instance, if a movie theater makes money from people attending the movies, the profits are dependent on the number of people that attend the movie. There are two things that change: the number of people and the dollars of profit. The number of people is the independent variable because it causes the profit to change; it determines how much profit will be made. Quite often, time or number of people are independent variables.

In the problems, the dependent and independent variables are not as clear as in other problems. It is harder to decide which causes the other to change. The number of garages seems to be the focus. As the table is being set up, the question that is asked is if there were 2 garages, how many blocks would be needed? This question makes the garages the cause of the change in number of blocks. If blocks were the independent variable, the question would be, "when there is one block, how many garages can be made?", and "when there are two blocks, how many garages can be made?".

There is a lot of information and mathematics in this workshop. It is tempting to emphasize the vocabulary, but the temptation is best resisted. Using technical vocabulary too soon can cause anxiety among the group, and inhibit learning. This seems especially true when the form of $y = 5x$ is used instead of $B = 5g$. This is not an exercise in learning vocabulary, but in experiencing hands-on activities that lead to the foundations of algebraic thinking.

Room Setup

- Desks or tables arranged in groups of 4
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda

Materials

Facilitator	Transparencies
• Overhead projector • Overhead pens • Transparencies, blank • Chart paper • Chart markers • Masking tape • Timer (optional) • Estimation questions (prepared by facilitator) • Inexpensive prizes • Unifix Cubes, Set of 20	*BLM 1: Welcome* *BLM 14: Searching for Patterns* *BLM 15: Garages* *BLM 18: Sample Garage Graph* *BLM 19: Connected Garages* *BLM 24: Models With Toothpicks*
Participant	**Handouts**
Individuals • Paper • Pencil • Reflection • Black Sharpie marker • Unifix Cubes, Set of 20 **Groups** • Chart markers	**One per participant for class** *BLM 14: Searching for Patterns* *BLM 15: Garages* *BLM 16: Garage Table* *BLM 17: Garage Graph* *BLM 19: Connected Garages* *BLM 20: Connected Garages Table* **One per group for class** *BLM 21: NCTM Algebra Standard* *BLM 22: NCTM Connections Standard* *BLM 23: NCTM Communications Standard* **One per participant for home (make packet for easy distribution)** *BLM 24: Models With Toothpicks* *BLM 25: List of Terms*

Timing

1 hour and 55 minutes

Preparation and Timing (1 hour and 55 minutes)

Part 1: Getting Started (10 minutes)

Set-up on tables:
Estimation activities, games, puzzles, or career activities for each group, snacks
Display transparency from workshop one:
BLM 1: Welcome
Make transparency of:
BLM 14: Searching for Patterns
Make copies for each participant:
BLM 14: Searching for Patterns

Part 2: Setting the Stage (20 minutes)

Distribute:
Materials listed under participant on previous page

Part 3: Garage Patterns (30 minutes)

Make transparency of:
BLM 15: Garages
Make copies for each participant:
BLM 15: Garages
BLM 16: Garage Table

Part 4: Recording Patterns (30 minutes)

Make transparency of:
BLM 18: Sample Garage Graph
BLM 19: Connected Garages
Make copies for each participant:
BLM 17: Garage Graph
BLM 19: Connected Garages
BLM 20: Connected Garages Table

Part 5: Standards (5 minutes)

Make copies for each participant:
BLM 21: NCTM Algebra Standard
BLM 22: NCTM Connections Standard
BLM 23: NCTM Communications Standard

Part 6: Connections (5 minutes)

No transparencies or handouts

Part 7: Take Home Applications (5 minutes)

Make transparency of:
BLM 24: Models With Toothpicks
Make copies and prepare a take home packet for each participant:
BLM 24: Models With Toothpicks
BLM 25: List of Terms

Part 8: Closing (10 minutes)

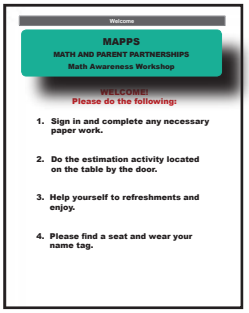
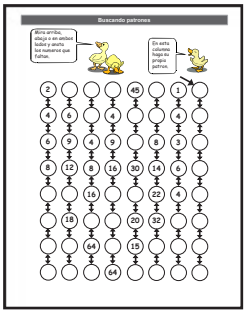
Prizes and reflection / evaluations

Facilitator Resources

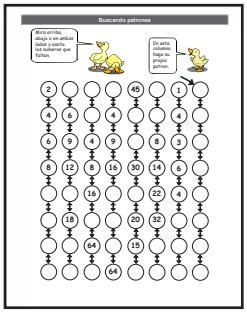
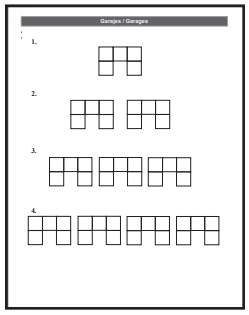
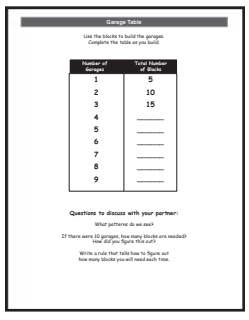
Books

Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, P. 222, 268. and 274, ISBN 0-87353-480-8, www.nctm.org

Activities

Preparation of Classroom	Notes
1. Arrange desks or tables in groups of 4-6. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up estimation activities. 2. Display the transparency of **BLM 1: Welcome!**. 3. On participant tables, distribute **BLM 14: Searching for Patterns,** pencils, paper, black permanent marker, and colored markers. 4. Prepare and display a poster with the agenda and purpose of the session.	BLM 1: Transparency  BLM 14: Transparency / Handout 
Part 1: Getting Started (10 minutes; parents and children)	
Searching for Patterns As the participants begin to arrive, direct them to the "Searching for Patterns" problem on their tables. Ask them to see if they can discover the rule and fill in the missing numbers. This problem is a good starter problem that the participants can work on individually or with their children. **Introductions** 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences. 3. Go over the agenda and purpose for the session. **Engagement** Tell the participants that they will be looking at patterns tonight as they create tables and graphs, and then they will be looking at the standards that have been set for teaching mathematics and discuss what has changed since the parents were in school.	
Part 2: Setting the Stage (20 minutes)	
1. The "Searching for Patterns" problem will help to prepare the participants for the activity that will be presented as well as set up the mood for the workshop.	

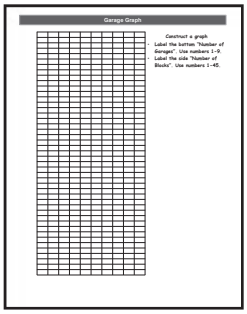
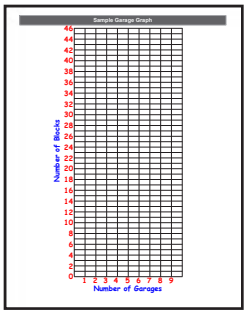
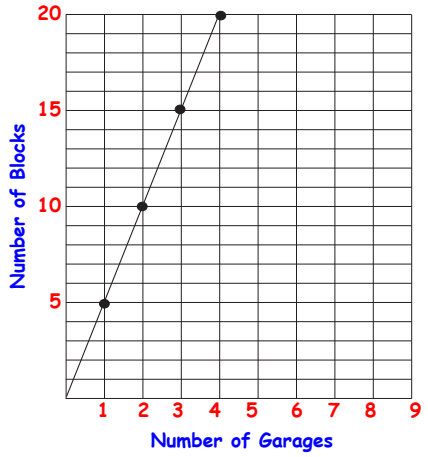
Activities

Part 2: Setting the Stage (continued)	Notes
2. The discussion of the problem is a good ice breaker. Give participants time to work on the paper. Then have them share some of the sequences that they have found. Have participants share the sequences that they made up with someone at their table, and then have each table share one sequence with the whole group. Display **BLM 14: Searching for Patterns** and have someone in each group write in the numbers of the sequence on the overhead that their group wants to share. Ask them to explain their patterns. 3. Tell the participants that looking at patterns is an essential part of mathematics. Much of what we know about mathematics today was discovered through examining patterns. If their children are good at looking for patterns, they will find mathematics easier to understand. Say: • *Now we will look at some patterns and see what we can do with them.*	BLM 14: Transparency / Handout 
Part 3: Garage Patterns (30 minutes)	
1. Distribute 20 unifix cubes to each participant. Display the transparency of **BLM 15: Garages**, revealing one sequence at a time. Reveal the first garage and say: • *This is how the elevation of a garage looks. An elevation is a drawing of an object that shows only one side or face. We would have to use 5 cubes to build this. Please build the garage with the cubes provided.* (Give participants a few moments to build the garage.) • *If we wanted to build two garages this is what it would look like.* (Reveal the line of the transparency that has 2 garages.) • *Please build the second garage. This is what three and four garages look like. Build the third and fourth garage.* 2. Allow participants approximately 2-3 minutes to complete building each of the garages. During this time distribute pencils, paper and the handout of **BLM 16: Garage Table** to each participant. Then say: • *You were given a "Garage Problem Table" to record the number of cubes needed for building the garages. Let's look at this table and complete it.* 3. Record the table on chart paper so that it can be referred to at a later time. Fill in the table as you get the responses to the questions. • *When we had one garage, how many blocks did we need?* (5 cubes)	BLM 15: Transparency  BLM 16: Handout 

Activities

Part 3: Garage Patterns (continued)	Notes
• *When we built two garages, how many blocks did we need?* (10 cubes) • *When we built 3 garages, how many blocks did we need?* (15 cubes) • *Take some time now to fill in your table and discuss the questions.* 4. Allow about 10 minutes for the participants to complete the table and discuss the questions with their partner. 5. Fill in the rest of the table on the chart paper by asking for volunteers for the answers. Then have volunteers share answers to the questions on their sheets. With a shy group, you might have to say that you saw an idea that looked like this (sharing an idea that you saw on someone's paper), and write the idea on the overhead for the group to discuss. It is not necessary at this time that they come up with an equation. If the participants have found an equation, have the participant share how they found it, and then try it with a few of the garage patterns to show how it works. • *How many blocks will we need to build 9 garages?* • *What patterns did you discuss with your partner?* • *What kind of rule or generalization were you able to write?* Some examples of written rules are: "I multiply the number of garages times 5" or "I keep adding 5" As participants share that the number of blocks is going up by 5 each time, record this on your chart so that the pattern becomes more visible. **See Note A.** The participants should come up with an equation as they look at the recordings on the table. Blocks = 5 times Garages, B = 5 times G, B = G times 5, B = 5 X G, or B = G X 5. Accept any correct version that is given to you. It is important not to take the leap from 5 times G to 5G, unless a participant volunteers it, and then be sure to explain that this is the algebraic method, showing the connection between 5G and 5 times G.	**A. NOTE:** It is important for participants to see the relationship between the two approaches: multiplying the number of garages by 5, and adding 5 each time. Recording the patterns on the table will help participants make this connection. **Number of Garages** / **Total Number of Blocks** 1 — 5 2 — (5) + 5 3 — (5 + 5) + 5 4 — (5 + 5 + 5) + 5

Activities

Part 4: Graphing Patterns (30 minutes)	Notes
1. Tell the participants that there is another way to record this pattern. Distribute **BLM 17: Garage Graph**. The participants will be constructing a graph with the number of garages as the independent variable on the x-axis and the number of blocks on the y-axis as the dependent variable. It is not necessary to get into a discussion about independent and dependent variables, however, if the subject comes up, have one of the participants volunteer an explanation. There is a definition in the list of terms that can be handed out now or at the end of the session. Say: *On the graph, lets label the axis. Across the bottom we will plot the number of garages" using the numbers 1-9. Be sure that the numbers fall right on the line and not between the lines. This should be labeled "Number of Garages" and numbered 1-9.* 2. Model this on the transparency of **BLM 18: Sample Garage Graph. See Note B.** Say: *Let's label the side"Number of Blocks" and then number from 1-46.* (Point to the vertical line on the graph.) *Now we can graph the numbers on our graph. For one garage we need 5 blocks, for two garages we need 10 blocks, for three garages we need 15 blocks. Please complete the graph on your own.* Allow 5 minutes for the participants to complete the graph. Circulate around the room and give assistance when needed. Ask: *What do you notice about the graph?* Participants should see that the points are all forming a diagonal line to the right and increasing at the same increments. Say: *The reason that the numbers we graphed are forming a line is because the numbers are changing at a constant rate. What is that rate of change? We can also use the graph to find information about our garages:* • *If I have 25 blocks I can make 5 garages, or* • *If I need to build 7 garages I need 35 blocks.* Model these points on the transparency. Model that the equation B = 5 times G produces the same results. Ask: *How might you use the blocks, the table, the rule, and the graph to find the number of blocks that are needed for 12 garages? Start wherever you want. After you find the answer one way, try another way to find the answer.*	BLM 17: Handout  BLM 18: Transparency  B. NOTE: See sample of Garage Graph below (showing the bottom half of the graph): 

Activities

Part 4: Graphing Patterns (continued)

3. Display the transparency of **BLM 19: Connected Garages** and distribute the handouts **BLM 19: Connected Garages** and **BLM 20: Connected Garages Table**. Ask:

- *How is this pattern similar to the first one?*
- *How is this different from the first group of garages?*

(Encourage volunteers to share their thoughts.)

In your groups, construct these garages and record your findings in a table. See if you can write a rule.

Give participants time to work on this and then ask for help filling in a table on chart paper. Ask them if anyone has found a pattern. **See Note C.** Remember to stay open to the variety of rules that could be written, and ask participants to try their rules for the different numbers on their table to see if it consistently works. A trap here is to have an answer in mind and not to see the possibilities of many rules that work. Some examples are:

- B = 2 + 3G
- B = 5 + 3 (G - 1)
- B = 3G + 2
- G = 1/3 (B - 2)
- B = 8 + 3 (G - 2)
- B = 3 (G + 1) - 1

4. As the participants see that it is changing by three, write the change out to the side on the chart. Show that this is a constant change and point out that it is similar to the change of five in the first garage problem example. Say:

Now, let's take the change backwards to 0 garages. Although this seems silly at first, it helps us to find a rule. The number of blocks will go down 3, and we will find our starting point.

Ask participants to use the table to make graph. Point out the starting point.

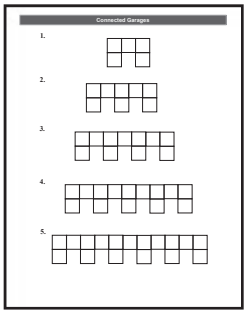
5. Ask:

How might you use the blocks, the table, the rule, and the graph to find the number of blocks that are needed for 12 garages? Start wherever you want. After you find the answer one way, try another way to find the answer.

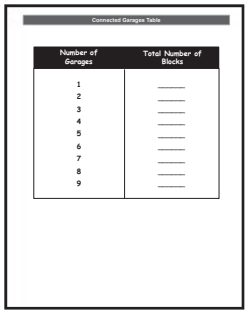
Have one person from a group share a method for finding the number of blocks for twelve garages. Have others continue sharing until all four methods (blocks, table, rule, and graph) are shared.

Notes

BLM 19: Transparency /Handout



BLM 20: Transparency

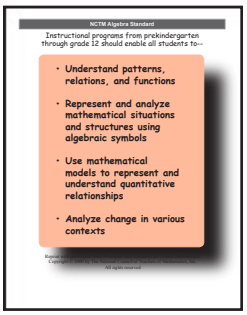
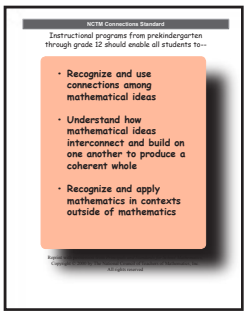
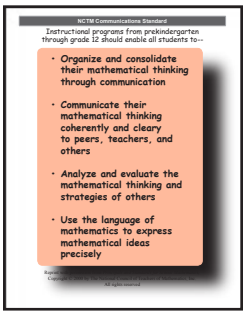


C. NOTE: Since this new rule is a 2-step equation, some participants will need help. They might see that the table shows a constant rate of change (3) but know that the rule is not multiply by three. Asking them if they can rewrite their # of blocks using the method of adding threes might help them arrive at a rule that either shows # of Garages times 3 plus 2 = # of blocks (3G + 2 = B), or even (# of Garages - 1) times 3, plus 5 = # of blocks (3(G - 1) + 5 = B).

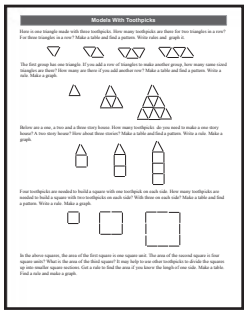
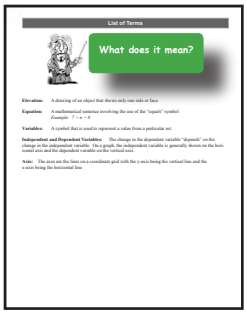
G	B
1	5
2	8
3	11
4	14

G	B
1	2+3
2	2+3+3
3	2+3+3+3
4	2+3+3+3+3

Activities

Part 5: Standards (5 minutes)	Notes
1. Tell the participants that it is time to look more closely at why they are doing the activities. Ask them to share at their tables how this is the same and how it is different from the way that they learned mathematics in school. After a few minutes of sharing with each other, ask for volunteers to share their thoughts with the whole group. 2. Explain that mathematics education is changing. It is driven by the desire to make the understanding of mathematics available to all students. Tell the participants that mathematics educators have a book of standards to guide them in teaching mathematics. They are each going to get some examples of the standards to look at and decide if it applied to this session. Each table is handed one of the standards to explore. **BLM 21: NCTM Algebra Standard, BLM 22: NCTM Connections Standard**, and **BLM 23: NCTM Communication Standard**. Give participants time to decide which of the items they did and to mark them with a check (not all of the items were done in the session). After a few minutes, have the groups report out.	BLM 21: Handout  NCTM Algebra Standard Instructional programs from prekindergarten through grade 12 should enable all students to-- • Understand patterns, relations, and functions • Represent and analyze mathematical situations and structures using algebraic symbols • Use mathematical models to represent and understand quantitative relationships • Analyze change in various contexts BLM 22: Handout  NCTM Connections Standard Instructional programs from prekindergarten through grade 12 should enable all students to-- • Recognize and use connections among mathematical ideas • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole • Recognize and apply mathematics in contexts outside of mathematics BLM 23: Handout  NCTM Communications Standard Instructional programs from prekindergarten through grade 12 should enable all students to-- • Organize and consolidate their mathematical thinking through communication • Communicate their mathematical thinking coherently and cleary to peers, teachers, and others • Analyze and evaluate the mathematical thinking and strategies of others • Use the language of mathematics to express mathematical ideas precisely
Part 6: Connections (5 minutes)	
1. Tell the participants that tonight they have used the exploration of patterns to practice three of the key elements of algebra: tables, graphs and rules (generalizations or equations). Research on how people learn has found that people need to make connections between what they know and the new thing that they are learning. During this session, you made connections about what you know about counting and patterns in searching for patterns, then you connected tables, graphs and equations to what you knew about the garage patterns. 2. Take some time to connect the session to your curriculum. Show the participants what their students are learning about patterns and how they are making connections in their everyday mathematics. 3. Ask participants how they might support their child's understanding of patterns. Have participants discuss it with their group. Have groups share their ideas.	

Activities

Part 7: Take Home Applications (5 minutes)	Notes
1. If time allows (for clarification purposes), display the transparency of **BLM 24:Models With Toothpicks** and go through questions. 2. Tell the participants that a great way to practice looking for patterns is to look at Hocus-Focus in the newspaper near the crossword puzzle. There is a new puzzle each day, and it helps us all look more closely at changes. 3. Distribute handouts of **BLM 24: Models With Toothpicks** and **BLM 25: List of Terms** for participants to take home.	BLM 24: Transparency /Handout  Models With Toothpicks BLM 25: Handout  List of Terms What does it mean?
Part 8: Closing (5 minutes)	
1. Distribute any prizes from estimations or drawings. 2. If your district does not have an evaluation form to use, you may want to use a reflection similar to *"Tonight I felt mathematically powerful when..."* or have them answer one of the following questions: • *What did you learn tonight?* • *What will you do with your son/daughter as a result of this session?*	