Distance Around

Investigating Pi

Outcomes

- To discover the value of pi using manipulatives and technology.
- To explore uses of pi, radius, diameter and circumference in real life situations.
- To build an awareness of how student-centered activities and discovery can be used to develop mathematical concepts.

Overview

As participants arrive, they are greeted and asked to do some measuring of circular objects that are placed around the room. The MAPPS program is described, and then the focus of the session: circles. Participants brainstorm where circles occur in everyday life and what measurements of circles can be taken.

More time is given for participants to take the measurements of the radius, diameter and circumference of the objects around the room. Then the groups share some of the measurements that they have taken. The group estimates the relationships among the three measurements. Then they calculate the ratio of the circumference to the diameter using calculators. Participants calculate the mean and the median of their information. They decide which measurement best represents their information. Then they look at the value of pi on the calculator to see how close they have come to its value.

Participants then spend some time making sense of the value of pi. They connect the value to "three and a little bit more". This helps the value of pi become more user friendly. Participants estimate several circumferences and diameters using the estimation of three and a little bit more.

If time permits, participants do the optional exploration of hat sizes and how the distance around the circumference of a tire relates to distance traveled and is used in determining the speed of the car.

Mathematics Background

The mathematical concept in this module is recognizing the relationship between diameter and circumference. Participants spend the evening using manipulatives to discover the ratio that is known as pi. It is meant to be an experience with constructivist teaching, having the participants construct the value of pi.

Meaning of Pi
The value of pi seems magical unless there has been some exploration that has led to the discovery of the value. When doing the measurements, participants will notice the larger the object (using the same unit of measurement), the closer their calculations will come to 3.14. When participants measure one of the round tables using centimeters, the ratio of the circumference to diameter comes very close to 3.14. Why is this so? Rounding off and human error in measurement lead to the variety in the ratios that participants will calculate. A 1 cm error on a 10 cm measurement is a 10% error. The same 1 cm error on a 100 cm measurement is only a 1% error. The relative size makes a difference and affects the accuracy of the ratios.

Making sense of numbers is a major part of mathematics. Pi is often perceived to be some unusual number. The fact that it is slightly more than three is often missed. This module attempts to make this connection for participants.

The Mad Hatter
This activity is an application of the ratio. The instructions ask participants to measure around their head and then look up their hat size on a chart. The ratio between hat size and the measurement around the head is: 3.14. It is true that heads are not perfectly round, but the ratio relates to the activity of the session and is a nice real-life connection.

Sidewall of a Tire
In this activity the number 65 refers to the height of the sidewall, or the Aspect Ratio. It is a percentage of the section width. In this example, you would take 65 per cent of 205 millimeters and this would give you the sidewall height.

Tire Applications
Some people have struggled with number 4. Here is another explanation of how it might be approached. The speedometer gets its information from the odometer, which is based on tire size (recommended size for the car). The odometer counts the rotations of the tire to determine the distance the car has traveled. If a car is going 60 miles per hour, it travels one mile (63,360 inches) each minute. Suppose a car is calibrated for the small tire. When the car travels 60 m.p.h., the small tire must rotate 877 times every minute (63,360 ÷ 72.22 = 877). It travels exactly one mile (877 x 72.22 in. = 63,360 in.). If the large tire was on the car instead, the car would have traveled 74,352 inches during the same minute (84.78 in. x 877 rotations). The speedometer would be fooled because it still would be counting 877 rotations as a mile, but only 747 rotations of the larger tire are needed to go a mile. Therefore, if the man was stopped for speeding and didn't know that he was speeding, he must have put larger tires on his car.

Room Setup

- Desks or tables arranged in groups of 4
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda
- Space to record explorations

Materials

Facilitator	Transparencies
• Overhead projector • Overhead pens • Overhead calculator (optional) • Transparencies, blank • Chart paper • Chart markers • Masking tape • Timer (optional) • Estimation questions (prepared by facilitator) • Inexpensive prizes	*BLM 1: Welcome* *BLM 52: Measuring Circles* *BLM 53: Ruler* *BLM 54: Recording Measurements* *BLM 55: Using Pi* *BLM 56: The Mad Hatter* *BLM 57: Sidewall of a Tire* *BLM 58: Tire Ad* *BLM 60: NCTM Measurement Standard and Expectations*
Participant	**Handouts**
Individuals • Paper • Pencil • Reflection • Calculators with π **Partners** • Circular items to measure • Measuring tape • Ruler (optional)	**One per participant for class** *BLM 52: Measuring Circles* *BLM 54: Recording Measurements* *BLM 56: The Mad Hatter* *BLM 59: Tire Applications* **One per participant for home** *BLM 61: Circumference Applications* *BLM 62: Facts Sheet: The Distance Around*

Timing

1 hour 55 minutes

Preparation and Timing (1 hour and 55 minutes)

Part 1: Getting Started (10 minutes)

Locate 10 circular items for participants to measure with partner
Display transparency from workshop one:
BLM 1: Welcome

Part 2: Setting the Stage (5 minutes)

No handouts or transparencies

Part 3: Measuring (30 minutes)

Make transparencies of:
BLM 52: Measuring Circles
BLM 53: Ruler
BLM 54: Recording Measurements
Make a copy for each participant:
BLM 52: Measuring Circles
BLM 54: Recording Measurements

Part 4: Finding the Ratio (20 minutes)

Use handouts from previous lesson

Part 5: Making Meaning of Pi (15 minutes)

Make transparency of:
BLM 55: Using Pi

Part 6: Applications (20 minutes) - Optional

Make transparencies of:
BLM 56: The Mad Hatter
BLM 57: Sidewall of a Tire
BLM 58: Tire Ad
Make a copy for each participant:
BLM 56: The Mad Hatter
BLM 59: Tire Applications

Part 7: Connections (5 minutes)

Make copies of specific lessons where the concept of pi is developed and applies in the curriculum of local district.
Make transparency of: (Optional)
BLM 60: NCTM Measurement Standard and Expectations

Part 8: Take Home Applications (5 minutes)

Make a copy for each participant:
BLM 61: Circumference Applications
BLM 62: Facts Sheet: The Distance Around

Part 9: Closing (5 minutes)

No handouts or transparencies
3-4 Inexpensive prizes for Estimation Question winners
Reflection / evaluations *(provided by the evaluation team)*

Facilitator Resources

Books

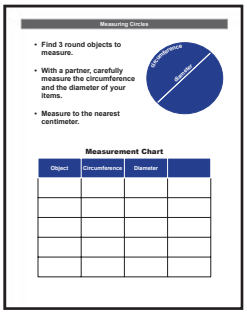
Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, P. 170, ISBN 0-87353-480-8, www.nctm.org

Curriculum Examples

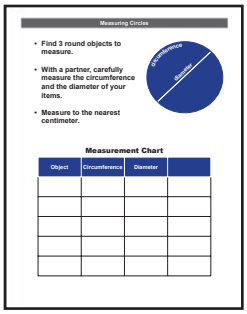
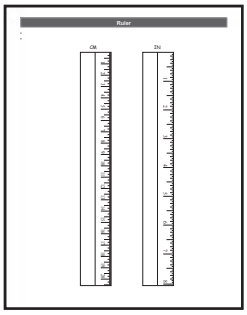
Everyday Mathematics: 5th grade, *Journal II*, Lesson 82, p. 320-322, study link 82.

Interactive Mathematics Program: Year 3: pages 59-123, *Orchard Hideout*, suggested transparency of the growth of the trees in the teachers' manual, p. 238-240.

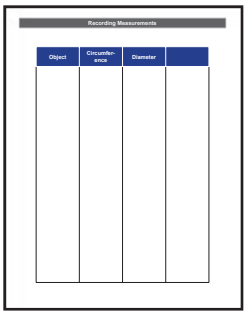
Activities

Preparation of Classroom	Notes
1. The estimation that is recommended for this workshop involves some preparation. Display 5 or 6 containers of various sizes that are all cylinders (Pringles can, coffee cans, soup cans, etc.). Number each cylinder. Then cut rectangles of colored construction paper that are the same size as the lateral surface area (like a soup label that fits around the can) for each of the cans. Glue then to a display and label them A, B, C, etc. Vary the orientation of the rectangles so that the height of the rectangles will not be as easy to match with the cans. Have participants guess which label would fit each can. 2. Arrange desks or tables in groups of 4-6. Set up a table with a sign-in sheet, name tags, and snacks. Distribute the handout of **BLM 52: Measuring Circles** to the participant tables. Have circular items set up around the room for measuring (have duplicates of items). After participants have completed the estimations, they can start measuring items around the room. 3. Display the transparency of **BLM 1: Welcome!**. 4. Prepare and display a poster with the agenda and purpose of the session.	BLM 1: Transparency  BLM 52: Transparency / Handout 
Part 1: Getting Started (10 minutes)	
Introductions 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences. 3. Give participants an overview of the session. Review the agenda and purpose of the session. Tell the participants that we will be exploring circles in this session. When discussing the agenda, let the participants know the plan for including children in the session.	
Part 2: Setting the Stage (5 minutes)	
1. Ask participants to think of examples of items that are circular that they use in everyday life.	

Activities

Part 2: Setting the Stage (continued)	Notes
2. Have each group decide on the one or two examples that they think are the best examples of circles in everyday life to share with everyone. Record the ideas on a blank transparency or on chart paper. Use one of the ideas as an example and ask what measurements could be taken. (Some ideas are the distance across, the distance around, the area, or half way across.)	
Part 3: Measuring (30 minutes)	
1. Display the transparency of **BLM 52: Measuring Circles.** 2. Tell participants that they will be exploring the relationship between the distance across the circle and the distance around the circle. If it has not already come up in the discussion, bring up the vocabulary of radius, diameter and circumference and illustrate them on the **Measuring Circles** transparency. 3. Model the measuring of the diameter and circumference on the circle on the **Measuring Circles** transparency. Also measure the distance across the circle on the transparency (or any item that you choose) and write it in the "Measurement Table". Then measure the circumference and write it on the tablet, modeling how you would like participants to fill out their table. It is helpful to use a transparency of **BLM 53: Ruler** to illustrate the measuring. At the same time, show the medal part of the measuring tape and tell how it is part of the distance measured. Show how the distance between 3cm and 4cm can be 3 1/2cm and where that would be on the ruler. 4. Have participants find a partner and start measuring items. Have them record their results on the Measuring Circles handout. Give participants 20 minutes for measuring items of their choice. Circulate among the participants to answer questions and check to see that they are understanding the instructions. (On occasion, there may be some participants that just guess at the measurements in order to save time. Help participants to measure items instead of guessing.) Tell them that we are going to do some explorations about relationships in a circle. Tell the participants: • *I would like to have each group (partnership) go and pick an item that you have already measured and bring it to your table.* **See Note A.**	BLM 52: Transparency / Handout  BLM 53: Transparency  **A. NOTE:** The goal is to have an odd number of items (it will be easier to find the median), and to have no more than 11 items that are recorded on the "Class Measurement Table". If you have a large number of participants each group of 4-6 should choose an item to report out. If you have a small number of participants, have groups report more than one object or have partners instead of groups report out.

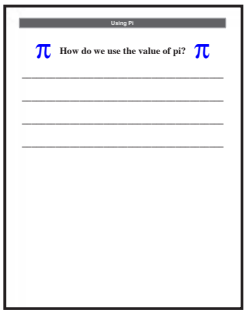
Activities

Part 3: Problem Solving (continued)	Notes
• *Measure your item one more time very carefully to check your work. Be ready to report your measurements.* **5.** Facilitators need to decide if they are going to accept incorrect measurements. It is recommended accepting them without comment and letting the participants come to the realization at some point that they measured improperly. This can be a touchy issue and must be handled in a nonthreatening manner. **6.** Display the transparency of **BLM 54: Recording Measurements**. As they finish, have partners (or groups) come up and write their measurements on the displayed transparency. Distribute handouts of **BLM 54: Recording Measurements**, and have the participants copy the list on their handout.	BLM 54: Transparency / Handout 
Part 4: Finding the Ratio (20 minutes)	
1. Estimating: Have participants look at the list on their **Recording Measurements** handout to find a relationship between the diameter and the circumference. Accept all ideas. The first sharing might be that the circumference is larger and the diameter is smaller. The object is to get them to realize that the circumference is approximately 3 times longer than the diameter. Questions that can help the discussion are: • *Is there something that can be done to the diameter to get to the circumference?* • *Will it always work? So, if we divide the circumference by the diameter, what should we get?* Ask how the calculators can be used to check this relationship. **2. Calculating the ratio:** Model the division of the circumference by the diameter, and then have participants use calculators to divide the circumference by the diameter of the item that they have at their table. The results should be rounded to 2 decimal places. Record the results on the table. Ask: • *What do most of these results have in common?* (It should be a 3) • *How can we find the number that best represents this list?* (Answers will vary, but average may be suggested.)	

Activities

Part 4: Finding the Ratio (continued)	Notes
3. Tell participants that one way to find a number to represent a group of numbers is to find the middle number, known as the median. 4. **Finding the median:** Have a participant help explain how to find the median of the numbers. The steps are: 1) Rewrite the numbers from smallest to largest. 2) Find the middle number (There will be a middle number if the number of items is odd, but there will not be a middle number if the number of items is even.) 3) If two numbers share the middle, find the number that is exactly halfway between them (If the middle numbers are 3 and 5, the median is 4). 5. **The ratio:** Record key concepts as in **Note B** while saying: • *What we have just found is a ratio. A ratio is a comparison of 2 things. We have just compared the circumference to the diameter. There is a name for this ratio. It is pi.* • *Pi is the most famous number in all of mathematics. The ratio of the circumference to the diameter is constant no matter what size the circle is! This is an amazing discovery that dates back to ancient times.* Have participants press the pi key to see what its value is and how close they got to that value. Have them discuss what might have made a difference in their computations. (The measurement might have been inaccruate, there may be a miscalculation or there might have been a mistake in rounding off.) Participants may want to see how well they did on their own list of items at this point. They should have a table with several items that they measured earlier. Give them some time to compute the ratio of the diameter to the circumference on their items and see how close they came to pi. It is important to take some time now for participants to make sense of what they have just discovered. The value of pi is just a little more than 3. This concept can be used to make easy estimations. Tell the participants that we are going to take some time to figure out what pi is all about. 1. Relating back to the model: Ask: *If I took the diameter and curved it around the circumference, how many times would it fit?*	**B. Note:** Record the main ideas on a transparency for the participants to take notes. Main ideas: Ratio: comparison of 2 things Pi (π): comparison of circumference to diameter - $\frac{\text{circumference}}{\text{diameter}}$

Activities

Part 5: Making Meaning of Pi (15 minutes)	Notes
Have participants try this with string. Have them cut the diameter and wrap it around the circumference of the item that they have at their table. *Approximately, what is the relationship?* You want them to get the idea that the circumference is a little more that 3 times the diameter. • *If I know the diameter, how can I estimate the circumference?* (Multiply by 3.) • *If I know the circumference, how can I estimate the diameter?* (Divide by 3) • *If the diameter is 5 inches, estimate what the circumference is.* (A little more than 15 inches) • *What if the diameter is 15 meters?* (A little more than 45 meters) • *If the circumference is 30 miles, estimate the diameter.* (A little less than 10 miles) Explain that they have been practicing using a benchmark. Many people think of π as a symbol instead of a number. Thinking of π as a number just slightly bigger than 3 helps students make sense of the relationship of the parts of a circle. 2. Display transparency **BLM 55: Using Pi**. Tell the participants that we have been working with the concept of pi. Ask if anyone can remember some of the formulas that use pi. As participants share, write the formulas on the transparency. **See Note C.** If the participants do not come up with the first three formulas in Note C, write the formulas for them. These formulas will be helpful when doing the applications, so display them and have the participants write them down. Show how each formula works using examples from **BLM 53: Recording Measurements**.	BLM 55: Transparency  **C. NOTE:** Brainstorm ways that pi is used. The group should come up with the following three formulas for area and circumference: 1. circumference = π • diameter 2. diameter = circumference ÷ π 3. circumference = π • radius • 2 4. diameter = 2 • radius (This information is by definition: we define the radius as one half of the diameter.) 5. * area = π • radius • radius * only write this formula for area if one of the participants brings it up. The module is about circumference, not area.

Activities

Part 6: Applications (20 minutes) - Optional

(If time permits, do part 6, if not, use the applications as a take-home activity.)

1. Display **BLM 56: The Mad Hatter**. Have participants measure their heads and figure out their hat size. If they take their hat size and multiply it by 3.14, they will be very close to their head measurement. Tell them that they might look into ring sizing: does it have the same relationship?

2. Display **BLM 57: Sidewall of a Tire**. Explain the meaning of r (radial tire), and the 13, 14 or 15 as the diameter of the wheel. Talk about the idea that when the tire is added to the wheel, the total diameter goes from one outside edge of the tire to the other outside edge.

3. Display the transparency of **BLM 58: Tire Ad**. Ask:

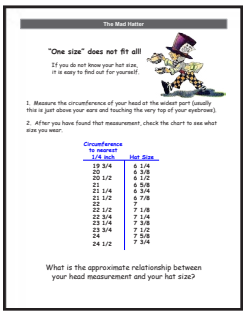
What do you know about tires from this ad?

Participants should discover that the last numbers represent the diameter of the wheel and that they are all radial tires. The may ask about the other numbers. For a discussion of what they mean and how to use them in calculations, refer to the mathematical background section of the module.

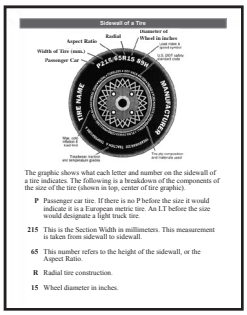
4. Distribute the handout of **BLM 59: Tire Application** and model finding the circumference and rotations of the first tire. An optional idea is to bring in a tire and mark a spot on it. Then roll it from one end of the room to the other, counting the rotations. Measure the distance. Another idea is for participants is to draw the tire on poster paper and then measure out the circumference with string. They can measure 3 rotations by laying the string out and measuring three lengths. Have participants work on questions 1-4. Process as you walk around, then have a reporting of discoveries. **See Note D.**

Notes

BLM 56: Transparency / Handout

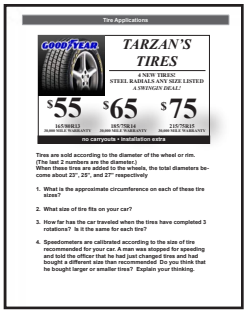


BLM 57: Transparency



BLM 58: Transparency

BLM 59: Handout



D. NOTE: Answers to questions on *BLM 59: Tire Application* handout.

1. $55: 23" x 3.14 = 72.2"
   $65: 25" x 3.14 = 78.5"
   $75: 27" x 3.14 = 84.8"
   (These round out differently using the pi key on the calculator)
2. This answer depends on the car. Have participants look.
3. To compute the distance, participants need to measure the diameter, multiply it by 3.14, and then multiply the new total by 3.
4. When a tire is smaller, it takes more rotations to go the same distance as a larger tire. The speedometer is related to how many rotations occur, the more rotations, the faster the speed. The man here must have put larger tires on so that the speedometer would register the car as going slower than it actually was.

Activities

Part 7: Connections (10 minutes)

1. Show specific lessons from middle and high school where the concept of pi is developed and applied. Some examples might include:
   - **Everyday Mathematics:**
     5th grade: *Journal II*, Lesson 82, page 320-322, study link 82
   - **Interactive Mathematics Program:**
     Year 3: pages 59-123, *Orchard Hideout*, suggested transparency of the growth of the trees in the teachers' manual, pages 238-240

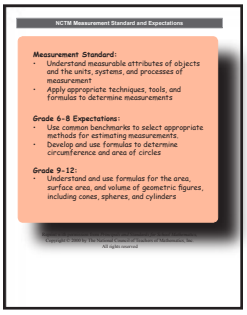
2. Distribute copies of activities from your curriculum that relate to these activities.

Optional: Standards

1. Show the progression of the standards in relationship to your state standards. The NTCM Standards relating to the understanding of circles is included here as an example.

2. Display the transparency of **BLM 60: NCTM Measurement Standard and Expectations**. Point out that students start working on the concepts of area and volume in the 4th grade and continue through high school.

BLM 60: Transparency

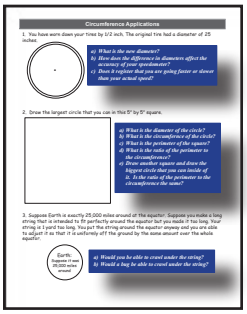


Part 8: Take Home Applications (5 minutes)

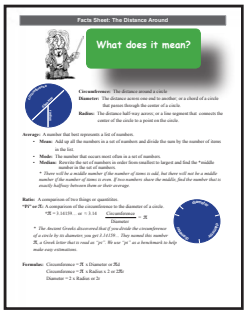
1. Distribute **BLM 61: Circumference Applications** and **BLM 62: Facts Sheet: The Distance Around**. If time, go through instructions.

2. Tell participants to find tire ad's in newspapers or junk mail at home and apply the principles they have learned.
   - *How do the measurements of bicycle tires vary from car tires?*
   - *If I have worn my tires down by 1 inch since I bought them, how will that affect my speedometer reading? Will it show me going faster or slower than my actual speed?*
   - *How do the sizes of rings relate to the circumference of your fingers?*

BLM 61: Handout



BLM 62: Handout



Part 9: Closing (5 minutes)

1. Distribute any prizes for Estimation Question winners.

2. If your district does not have an evaluation form to use, you may want to use a reflection similar to "*Tonight I surprised myself when I was able to . . .*"