SESSION ONE
GEOMETRY WITH TANGRAMS AND PAPER

Outcomes

- Develop confidence in working with geometrical shapes such as right triangles, squares, and parallelograms represented by concrete pieces made of cardboard, plastic, or paper
- Introduce basic geometrical vocabulary used in the context of tangram puzzles and patty paper.
- Establish relations between the areas of the pieces used.

Overview

In the first session on geometry it is important that participants realize that geometry can be dealt in very concrete terms and that they do a lot of geometrical thinking in tasks that are usually not thought of as geometry. The use of tangrams allows them to develop confidence and sets the stage to the gradual development of geometrical ideas. In the beginning it is convenient to use informal terms in conjunction with the more formal geometrical vocabulary.

Time

10–15 minutes	In the opening activity participants have the opportunity to solve tangram puzzles. They use right triangles, squares, and parallelograms to fill in given outlines and form shapes similar to given silhouettes of different animals.
30–40 minutes	Participants explore systematically with tangram pieces to form squares and other shapes with a given number of pieces.
25–30 minutes	Participants will use right triangles of two sizes and a square to explore relations between their areas. This will give them an idea of the kind of geometry young children can do using concrete pieces.
15–20 minutes	Patty paper that comes in square sheets is very convenient to explore empirically properties of squares. Participants fold and overlap to determine congruency of the different parts of the square.
5–10 minutes	Participants will fold a two smokestack ship using origami paper. The folds will form geometrical shapes. Participants can see relations between the sizes of the areas of different shapes.

Materials

Facilitator	Transparencies (Eng. & Spanish)
• A set of tangrams for the overhead • Large squares of paper for demonstration (tamale paper works fine) • Cardboard - to make triangles and squares from BLM 5 (facilitator makes these before class)	*BLM 9.1-2: Van Hiele Levels*

Materials

Participant	Handouts (English & Spanish)
• One tangram set per participant • Origami paper - one square sheet per participant • Patty paper - several squares per participant (one box with 1000 sheets will be enough for all sessions)	**Facilitator prepares one set per participant before class** *BLM 5: Hands-On Geometry: Square and Triangles* **One per participant for class** *BLM 1: Tangram Puzzle Pieces* *BLM 2: Shape of the Bird* *BLM 3: Further Activities with Tangrams* *BLMs 4.1-3: Geometry Explorations with Tangrams* *BLMs 6.1-6: Hands-On Geometry: Activities* *BLMs 7.1-3: Verifying Properties of Squares* *BLMs 8.1-2: A Two-Smokestack Ship* *BLMs 10.1-2: Van Hiele Levels of Development in Geometry* **One per participant for home** *BLMs 11.1-2: Tangram Puzzle Solutions*

Activities

Preparation of Classroom	Notes
1. Prepare handouts for distribution for class activities. 2. Have participant materials, manipulatives, and handouts on the tables. 3. Make a packet of BLM 11.1-2: **Tangram Puzzle Solutions** for participants to take home at the end of the session. Have a set of tangrams on the tables for each participant. 4. Prepare and cut cardboard square and triangle figures from BLM 5 for **Hands-on Geometry** activities.	Handouts can either be placed on the tables before the session begins or passed out at the beginning of each activity.
Getting to Know Each Other (10-15 minutes)	
1. Introduce yourself and tell a little bit about your professional and personal background. If you have children be sure to talk about them. Parents will feel comfortable relating to you as a fellow parent. 2. Tell participants that this course is about geometry and is designed to serve three purposes: • To expand their understanding of mathematics • To experience mathematical activities that can be used with their children at home • To have fun! 3. Tell participants that the course will be more enjoyable if we get to know one another and learn each other's names.	

Activities

Getting to Know Each Other (10-15 minutes)

- Give each participant an index card.
- Ask each participant to fold the card to form a tent.
- Ask them to write their name in large letters and to write the first name and the grade level of each of their children on the other side of the card.

4. After the name cards are completed, ask participants to introduce themselves and share how many children they have in school.

Notes

Tangram Puzzles (10-15 minutes)

Materials and handouts:

- Tangrams
- BLM 1: Tangram Puzzle Pieces
- BLM 2: Shape of the Bird
- BLM 3: Further Activities with Tangrams

Opening Activty

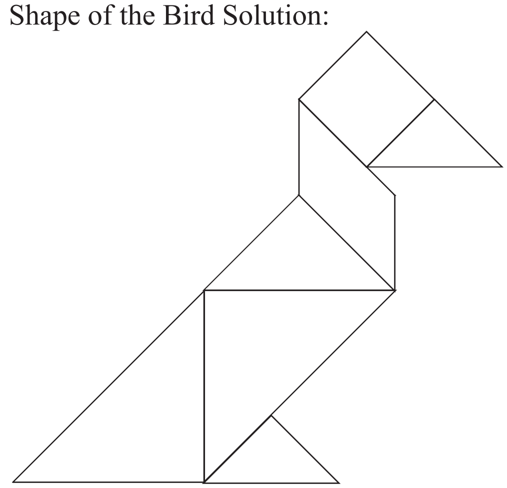
1. A tangram set is formed by seven pieces. These are provided for participants to do the activity. Participants (the same as children) should have the opportunity to play first with the tangram pieces, forming puzzles such as the one shown below. By forming puzzles participants will familiarize themselves with the geometrical shapes that form part of the tangram set. They will also learn how some of the pieces combined can form other shapes. They will also develop the ability to see geometrical shapes embedded within other shapes. Tangrams can be used to develop an informal mathematical environment.

2. Ask participants to use the seven tangram pieces to form the shape of the bird on the handout. They should try this on their own before looking at the solution at the end of the session. The instructor may want to help participants that seem to be stuck by indicating how to place one of the pieces. See solution to puzzle in notes.

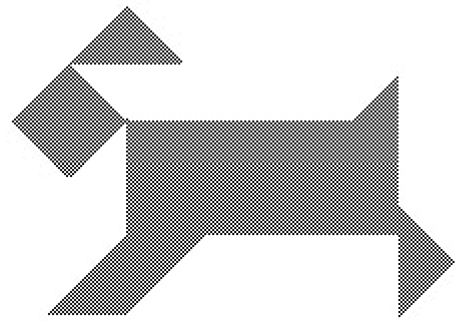
For home, participants can copy the **Tangram Puzzle Pieces** in the handout, and paste them onto cardboard and make their own set. Tangrams are also available commercially and they are quite inexpensive (an individual set can be ordered for about $1 from companies like Nasco).

These activities are the same kind that participants can do with their own children. Participants should have the opportunity to try these activities first.

Shape of the Bird Solution:



Activities

Tangram Puzzles (continued)	Notes
Optional Activity (for those who finish early) As participants get more familiar with the tangram pieces they can also try to do designs shown as a silhouette, but at a different scale as shown in the additional activities	Silhouette: 

Geometric Explorations with Tangrams (30-40 minutes)	Notes
Materials and handouts: • Tangrams • BLMs 4.1-3: Geometry Explorations with Tangrams	
Activity 1 1. Ask participants to fill in the outline for the big square tangram (all seven pieces are used). 2. This activity can be quite challenging by showing only the outline of the solution, so that the participants have to figure out where the different pieces go inside. Let participants fill the square themselves without looking at the solution at the end of the handout. 3. Participants should notice that the parallelogram is the only piece in the tangram set that is not symmetrical. Therefore, sometimes they will have to flip their parallelogram to fit into a given parallelogram space.	Participants may want to know that for small children in kindergarten or first grade, an appropriate activity would be to provide the outlined solution as shown on the solution page and let them fill it with the pieces.
Activity 2 Ask participants to form the shapes in Activity 2 handout using only the medium triangle and the two small triangles (the shapes are not shown at their actual size). Ask them to try on their own before looking at the solutions. Ask participants to report their solutions on the grid	Activities for upper elementary children (fourth and fifth graders) may include change of scaling. Children are given the shape to be built, but at a different scale. That is, the shape shown is not of the required size.
Activity 3 1. Ask participants to find whether they can form a square with exactly: a) one piece b) two pieces c) three pieces	This activity is appropriate for students in 4th - 5th grade.

Activities

Geometric Explorations with Tangrams (continued)	Notes
d) four pieces e) five pieces f) six pieces g) seven pieces 2. For each case ask participants to find as many solutions as possible. Ask them to use the grid to record their solutions. 3. All problems can be solved with one set of tangrams, except for forming the square with six pieces. To solve this problem, participants can use six pieces from two sets of tangrams.	Tangrams can also be used to develop problems where more than one solution is possible, or where a solution is possible only if we modify the condition of the problem.
Hands-On Geometry in the Early Grades (25-30 minutes)	
Materials and handouts: • Cardboard figures for Hands-On Geometry • BLMs 6.1-6: Hands-On Geometry 1. These activities are examples of creative, informal, intuitive geometry activities to stress higher-order thinking with young children. With these activities children develop spatial concepts such as conservation of area —if we cut a figure into two parts and rearrange the pieces we will obtain another figure that has the same area, even though it may look bigger. Participants will get a better feeling of what kind of geometrical activities young children can do. 2. Informal language such as “the small triangle” can be used and accepted at this level. The number of activities can be increased for higher grades. Activities can be adjusted for each grade with more vocabulary added, and a more precise description of the figures. For example, first graders may say “a square has four equal sides and four equal angles.” Quantitative relations among figures can be made explicit later. For example, second and third graders may say “the area of the square is twice the area of the triangle”. For the activities, right isosceles triangles of two sizes, and a square cut out from cardboard as in Figure 1 of the handout are given to the participants. 3. In these activities a special kind of triangle is used. Two of the sides of the triangle are congruent, and it has a right angle. Participants should realize that these are not	Participants can first do these activities themselves. Later, participants can do these activities with their own children. Children in kindergarten can do the first five activities in a session of 20-30 minutes. It is important that participants feel free to explore and experiment with geometrical figures in the same way young children would do using concrete tools such as geometric shapes made of cardboard that can be turned and moved. The experimental phase in geometry is crucial for all people in their development of geometrical thinking. The activities described here use the same kind of geometrical shapes used in tangrams. These activities are a natural sequence to the tangram activities. The emphasis however this time is not in just filling outlines, but making explicit relationships between parts of the different geometrical figures.

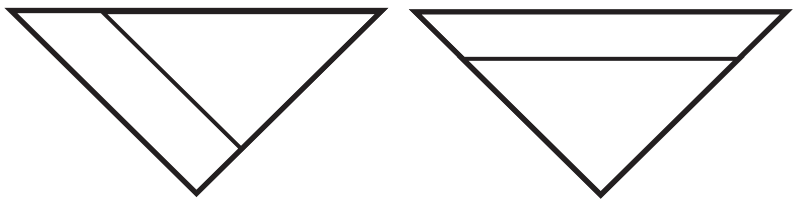
Activities

Hands-On Geometry in the Early Grades (25-30 minutes)	Notes
the only triangles that exist. It would be good to show a few examples of triangles so that participants do not over generalize the results from these activities to all triangles. 4. It is not necessary to do all the activities. Some people need more time. It is preferably to allow them to explore and experiment at their own pace rather than rushing them to try to cover many activities. Some activities build on the previous ones. Others can be skipped without affecting the rest (activities 4 and 7 are optional).	
Activity 1 1. Ask participants to take the cardboard square from Figure 1 and cover the square in Figure 2, to see that both squares have the same size and same shape. Both squares have the same area. Ask participants to use the cardboard square and cover Figure 3, to see that it is also a square and has the same shape and size as the first.	In kindergarten, most children are familiar with the shape of a square and are able to recognize and correctly show squares in their classrooms (calendar, boxes, frames, tiles, etc.).
2. At the first level of development of geometrical thinking, people focus on geometrical shapes based on their appearance. At this level, a square, when it is rotated, does not look the same, and people perceive it as a diamond instead. In many contexts other than geometry, when an object is placed in a different position it becomes indeed a different object. Consider for example the letter **b**, when it is turned 180°, it becomes **q**, which is a different letter. In geometry however, a figure is defined by intrinsic characteristics such as the lengths of sides and angles. Also, the relative position to the borders of the page is not important. The first activity will help people at the first level of geometrical thinking realize that the shape in the second orientation is indeed a square.	Sometimes children call Figure 3 a diamond, or a kite, and some emphatically say that it is not a square. However, after the activity children agree to call it a square, since it was clear that it was the same shape and size as the cardboard square and as the square in Figure 2. They recognize that it is the same shape, turned.
Activity 2 1. Ask participants to take one of the small cardboard triangles from Figure 1, and cover the triangle in Figure 4 to see that they are the same shape and same size.	This activity allows children to see that position and orientation of triangles do not alter congruent triangles.
2. Next, ask them to cover each of the triangles in Figure 5 with the cardboard triangle. This activity helps people see that congruent triangles in different positions are indeed the same. Using the same cardboard triangle to	Participants should realize that some children will rotate the background, that is, the figures on the handout rather than the cardboard triangles. Of course these alternative strategies are also valid.

Activities

Hands-On Geometry in the Early Grades (continued)	Notes
cover successively each of the triangles will convince them that although the triangles on the page look different they are indeed congruent to the same triangle.	
Activity 3 1. Ask participants to use two of the small triangles from Figure 1 to cover the square in Figure 2. Ask participants to use the same two triangles to cover the square in Figure 3. Participants (and children) can do the activity of forming the square (in the two positions) with two triangles with relative ease. In this case matching congruent angles or matching congruent sides lead to the solution.	For second and third graders, one goal is to see that the amount of cardboard needed to make the square is twice as much as the cardboard needed to make a single triangle, and that the area of the square is twice as big as the area of that triangle.
2. People use different strategies to form the same square in different positions with the two small triangles. Some people take the two small triangles together and translated and rotated both at the same time to fit into a new figure. Some people reassemble each figure, piece by piece each time, some of them always using consistently the same method, some of them using trial and error each time.	
Activity 4 (optional) Participants can take the big cardboard triangle from Figure 1, and cover each of the triangles in figure 6 with it.	The purpose is to reinforce what they experienced in activity 2, that the position and orientation of the triangle can change; this time bigger triangles are used. The strategy a person used may give a hint as to what type of mental picture and operations the person is using.
Activity 5 1. Ask participants to use two small triangles from Figure 1 to cover each of the triangles in Figure 6.	For second and third graders, a goal is to see that the area of the big triangle is twice as big as the area of the small triangle.
2. Forming the big triangle with two small triangles can take more time for some people than forming the square with two triangles. The instructor should not solve the problem for them, but provide a hint as to how to accommodate one of the triangles so that the person can fit the other one. It is sometimes surprising for adults—participants and teachers alike— to realize that these activities can be quite difficult in the beginning. They will thus realize that for some children these activities may not be easy.	In second and third grade, after activity 5, children can be asked to look at the big triangle and the square in Figure 1. Ask them whether the square or the big triangle would require more cardboard to be made (has more area). Some children may answer that “the triangle, because it is bigger”. This answer reflects that some students still focus only on one aspect of the figures, their longest side, to compare the amount of cardboard needed, rather than comparing the areas. It is important that we help children move beyond those limited conceptions.

Activities

Hands-On Geometry in the Early Grades (continued)	Notes
Some will need to be helped, because they use a dead-end strategy, matching corresponding angles from small and big triangles, as in the next figure. 	
Activity 6 1. The purpose of this activity is to focus on comparing the area of the square and the area of the big triangle not just by their appearance. Ask participants to compare the area of the square in Figure 7 with the area of the big triangle in Figure 8. Ask them to cover the square with two small triangles, and use the same two triangles to cover the big triangle, to see that the area of the square is equal to the area of the big triangle. Realizing that both were formed by two small triangles, participants will agree that the two shapes need the same amount of cardboard (have the same area). Although the square and the big triangle are not congruent and different in shape, they have the same area.	After doing the activity most children realize that the area is conserved if a figure is cut out and the pieces rearranged.
2. Participants need to realize that even after the activities of forming the square with two small triangles, and using the same two triangles to fill the bigger triangle, some children still perceive the triangle to have more area or cardboard.	Some children are mislead by the fact that the longest side of the triangle is longer than any dimension of the square. This can be an indication that the child is still working at the first level of geometrical thinking, where the appearance of a figure is the main factor.
Activity 7 1. Ask participants to cover the parallelograms in figure 9 with two small triangles from Figure 1, to see that these shapes have the same area as the square. 2. Even though parallelograms may not be familiar for many people, forming the parallelograms with two triangles is usually easier for them than the activity of forming a big triangle with two triangles. Part of the reason may be that in the case of the parallelogram, matching angles leads to a solution half of the times.	Forming the parallelograms with two triangles is usually also easier for students than the activity of forming a big triangle with two triangles.

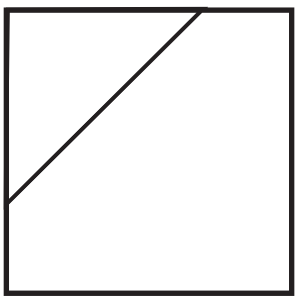
Activities

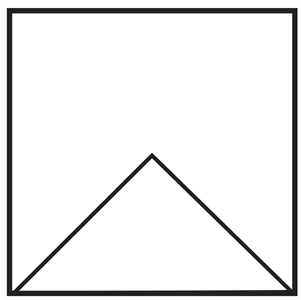
Hands-On Geometry in the Early Grades (continued)

Activity 8 (optional)

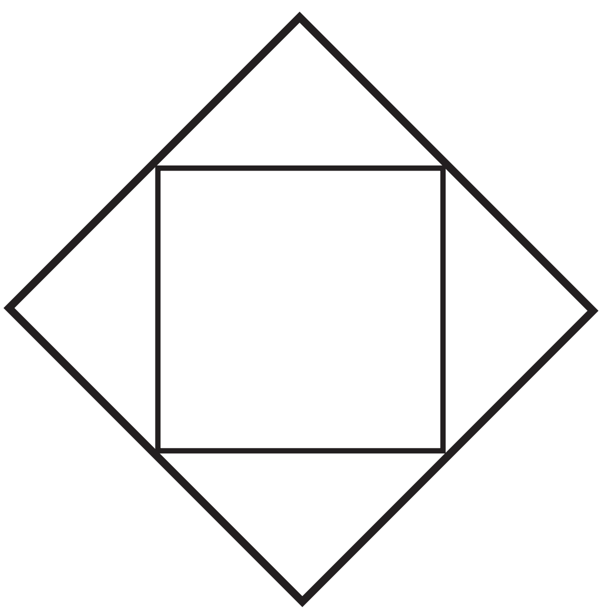
1. Ask participants to cover the square in Figure 10 with four small triangles from Figure 1, and then the square in Figure 11.

2. Forming the big square with the four triangles is challenging for most people, although some find the solution very quickly. Some of the people need to be helped because they insist in matching right angles (left figure below), rather than trying to match congruent sides to get the solution (figure on the right).





3. When participants are not given the frame in Activity 8 to solve the problem "to form a big square with four triangles", they usually will need more time to solve the problem. The absence of the frame offers the opportunity for some people to find a different solution (see Figure).



Notes

Participants may be surprised by the difficulty that some of them had in forming the figures with the small triangles. This will help them understand how difficult it can be for young children.

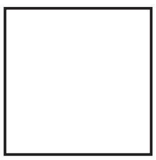
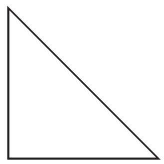
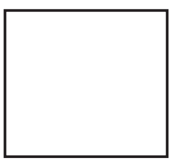
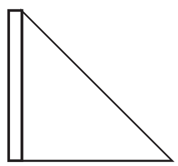
Activities

Hands-On Geometry in the Early Grades (continued)	Notes
Activity 9 1. Ask participants to compare the area of the two small squares with the area of the big square in Figure 12. Ask them to cover each small square with two small triangles from Figure 1, and use the same four triangles to cover the big square. Participants can see that the sum of the areas of the two small squares is equal to the area of the big square, and that the area of the big square is twice as big as the area of the small square. 2. Some of the people have difficulty in forming the bigger square with the small triangles. Rather than giving the whole solution away, the instructor may want to give them some hints as to how one or two of the pieces fit. The instructor may emphasize that the side of the bigger square is congruent to the diagonal of the smaller square.	
Activity 10 1. Ask participants to use the four small triangles of Figure 1 to cover the two small squares in Figure 13. Then, with the same four triangles ask them to cover the big square. In this figure the sum of the areas of the two small squares is equal to the area of the big square. They may recognize a special case of the Pythagorean Theorem. They should notice the triangle in the center is a right triangle; that the big square is the square on the hypotenuse of the triangle; and that the other squares are the squares on the legs of the right triangle. The sum of the areas of the squares on the legs of a right triangle is equal to the area of the square on the hypotenuse. 2. The instructor can focus the attention of the participants on the triangle among the squares. They should see that it is a right triangle. They should notice that each square has a side that is precisely equal to the corresponding side of the triangle. They should also be able to express that the sum of the areas of the squares on the two smaller sides of this right triangle is equal to the area of the square on the longer side. In a later session participants will see that this result (Pythagorean Theorem) is valid for any right triangle, not just an isosceles right triangle. It is convenient, however, for people to get familiar with the theorem in special cases before dealing with the general case.	Participants need to realize that children may use different strategies to form the same figure in different positions with the two triangles from Figure 1. Some children rotate the background, that is, the figures on the handout rather than the cardboard triangles. Some children take the two small triangles together and translate and rotate both at the same time to fit into a new figure. Some children reassemble each figure, piece by piece each time, with some of them always using consistently the same method, and others using trial and error each time. It is important that we allow students to use their own methods.

Activities

Verifying Properties of Squares (15 - 20 minutes)	Notes
Materials and handouts: • Patty Paper • BLMs 7.1-3: Verifying Properties of Squares 1. An important step in the development of geometrical thinking is to pass from mere perception to empirical verification of properties. Rather than just saying "it looks like they are equal", participants (and students) should be encouraged to compare lengths and angles by overlapping or by measuring. They are learning to seek and provide evidence for their assertions. 2. There are two important differences in the thinking of students or participants who are at the first level of development of geometrical thinking and those who are at the second level. One difference is that at the first level the shape is considered as a whole; the person does not focus attention on parts of the figure such as sides or angles. At the second level, it becomes meaningful for the person to talk about parts of the figure as objects of interest in their own. 3. The second difference is that at the first level appearance of shapes is very important. If two things look the same then the person considers them to be the same, there is no felt need for verification. At the second level, the person is willing to measure or to verify empirically by overlapping to make sure that two sides or angles are indeed equal. 4. The main goal of this set of activities is to help participants develop the ability to verify empirically that the square has indeed the properties it seems to have. When we look at a square, adjacent sides seem to be of equal length. We can use a ruler to measure both sides and verify whether that is the case. Another practical method is to have shapes made of paper and fold the shapes to overlap the sides or angles that we want to compare. Activities like these will help participants develop as a habit of mind to verify whether shapes, lengths, or angles, that look the same are indeed the same.	

Activities

Verifying Properties of Squares (continued)	Notes
5. In the case of squares, patty paper is very convenient because some brands come in squares. A good size is 5 1/2 by 5 1/2 (make sure the paper is really square, because some brands come in a rectangular shape that is not quite square). One way to verify that it is really a square is by folding through a corner so that two adjacent sides overlap, and see whether the crease does in fact pass through the opposite corner.   Square crease goes through opposite corner   not quite square crease does not go through opposite corner	For some of the participants who are at level 1 and do not see the need to empirically verify properties, a good experience is to give them a shape that looks like a square but that is not quite a square. It is not until they fold it that they see that sides they thought were equal are indeed slightly different.
6. It is important to make explicit what information we can obtain about properties of a square by doing a certain fold. Often more than one property can be verified with the same fold. For example, when folding along one diagonal, we can see that two pairs of adjacent sides are equal; that a pair of opposite angles are equal; and that the diagonal bisects the angles.	
Activity 1 1. Ask participants to fold the square in half, where the crease is parallel to two sides. Ask them about the size of opposite sides. Ask them about the size of consecutive angles. 2. One pair of opposite sides will match each other, so that participants can see that they are indeed congruent. At the same time, they can see that two pairs of consecutive angles also overlap, showing that the angles are also congruent. The instructor can help participants see how the mid parallel is a line of symmetry.	Folding the square along the mid parallel usually does not present problems for participants. However they need to be made aware of what properties of the square are verified by this process.

Activities

Verifying Properties of Squares (continued)	Notes
Activity 2 1. Ask participants to fold the square along a diagonal. Folding the square along the diagonal may be a little more difficult for some participants. One way to fold is by overlapping opposite corners, hold them tightly together and ensure that the crease does indeed go through the other two corners. Another method is to slide one side so that it overlaps the adjacent side and make sure the crease goes through the corner formed by those two sides and the opposite corner. 2. Ask participants about the size of opposite angles. Ask them about the size of consecutive sides. 3. When the square is folded along the diagonal, participants can see that two pairs of adjacent sides overlap exactly, so they are congruent, and that one pair of opposite angles overlap exactly, so they are congruent. 4. At this point, the instructor may guide the discussion so that participants use the information from the first two activities to infer that all four sides of the square are congruent and all for angles are also congruent. Some participants will use additional direct verifications. They may fold along the other mid parallel to show that the other pair of opposite sides is indeed congruent, or fold along the other diagonal to show that the other pair of adjacent sides is congruent. 5. Other participants may use logical inference, without having to verify again. For example, if they have already convinced themselves that side a is congruent to side **d**, and side a is congruent to side **b**, then some will see that side **b** is congruent to side d. This represents a more advanced level of geometrical thinking, one that is commonly found at level 3.	It is convenient that participants use different sheets of patty paper for each of the activities. That way previous creases will not distract them
Activity 3 (optional) Ask participants to find the center of the square by folding the two mid parallels. Four small squares will be formed. Ask them to compare the area of one of these small squares to the area of the original square. Let them compare the length of the sides of the small squares and the original squares. Ask them about the angle formed by the two mid parallels.	Activities 3 to 7 are optional. For activities 3 to 6, some of the same results can be found later when folding the two-smokestack ship. The results in activity 6 can be found in a later session using pattern blocks.

Activities

Verifying Properties of Squares (continued)	Notes
Activity 4 (optional) 1. Ask participants to find the center of the square by folding the two diagonals. Ask them to verify that in a square the diagonals cut each other in half. Ask them to verify that the diagonals of a square intersect each other at a right angle. 2. The two diagonals form four right triangles. What can they say about the area of these four triangles? Let them compare the area of one triangle to the area of the original square. 3. Ask them to verify that the diagonals and the mid parallels cross indeed at the same point. **Activity 5 (optional)** Ask participant to compare the area of one of the triangles formed by the two diagonals with the area of one of the small squares formed by the to mid parallels. **Activity 6 (optional)** Ask participants to find the center of the square by folding the mid parallels. Now fold the corners of the square so that they meet at the center. Four right triangles and a square are formed. Compare the area of this square to the area of the original square. **Activity 7 (optional)** 1. Explain that mathematicians have agreed that the measure of a right angle is 90°. 2. Ask participants to verify that two right angles put together form a straight line. Therefore, the measure of a straight angle is 180°. 3. Ask participants to verify that four right angles put together fit exactly. The full turn is therefore 360°.	

Activities

Connections: A Two-Smokestack Ship (5 - 10 minutes)	
Materials and handouts: • Orgami Paper • BLMs 8.1-2: A Two-Smokestack Ship	
A Two-Smokestack Ship 1. Participants should fold the two-smokestack ship themselves, so that later they can guide their own children in folding such a ship. Participants should be allowed plenty of time at each step, so that nobody falls behind. The instructor may use the waiting time to emphasize mathematical aspects of the figures obtained at each step, while those who fold quickly wait for everybody to finish the corresponding fold. The instructor needs to wander among the different tables to make sure everybody is folding in the same way. 2. Facilitators can enrich the vocabulary of participants by using geometrical expressions as they instruct participants how to fold. A web site with color pictures of all the folds has been developed to remember the process. It can be accessed at: http://www.public.asu.edu/~aaafp/barco/smkst.html	Activities that involve folding paper to make ships and planes offer the opportunity for young children to develop intuitions and familiarity with geometric situations. Later, children can relate geometrical ideas to those previous experiences. In this activity participants will have the opportunity to fold a ship themselves. Later they can conduct their own children through the activity.
Folding a Two-Smokestack Ship 1. Start with a square sheet of orgami paper. 2. Let participants find the center of the square by folding the middle lines and opening the sheet again. 3. Participants will notice that four squares are formed. Each square has an area that is 1/4 of the area of the original square. 4. Ask participants to fold one corner so that the vertex is on the center (fig. 3). They obtain a right triangle. 5. Ask them to fold the other three corners (fig. 4). They obtain one square formed by four right triangles. The area of this square is half the area of the original square. 6. At every step the instructor can model the use of mathematical terms to describe parts of the folded figure. Although many participants have experiences doing paper folding crafts, very seldom are these used with mathematics in mind.	Participants can also find the center by folding the diagonals, but for the ship it is more convenient to use the middle lines.

Activities

Connections (continued)	Notes
7. Part of the mathematical vocabulary that can be used to describe the figures in each step is incorporated in the participants' activity. During the class the instructor may want to incorporate additional terms, or make properties of figures explicit. For example, when starting with the square, remind participants that all four sides are of equal length (congruent) and that all angles are 90˚. Also that the line obtained by folding the square in two equal parts is parallel to two of the borders, and that the second fold will be perpendicular to the first one. 8. In addition to noticing the relationships between the areas of the shapes obtained in successive steps of folding the ship, the instructor can help participants provide different kinds of arguments of why these are true. For example, in the case of figure 2, participants can see that there are four equal squares; therefore each one is 1/4 of the total. They may also say that the length of the side of the smaller square is 1/2 the length of the original square. Its area can be computed by multiplying 1/2 x 1/2, and that is equal to 1/4. 9. For figure 4, participants can see that the area of the folded square is one half the area of the original square in different ways. One way is to see that there are two sheets of paper covering the square (a square on one side and four right triangles on the other). They can also open the triangles and see that eight equal triangles were formed, and that there are four triangles inside the square and four outside. 10. Figure 4 is an opportunity to help participants see that even though it is tilted and looks like a diamond, the figure is still a square. Indeed, because participants will have the diamond in their hands, they will be able to rotate it so they can see that indeed it is a square. 11. Ask participants to flip the folded paper so that the other side is up. Ask them to fold one corner to the center (fig. 5). 12. Then they should fold the other corners to the center (fig. 6). They again obtain a square. The area of this square is half the area of the previous square. Its area is therefore 1/4 the area of the original square.	For many participants, the vocabulary may be new, and for them the paper folding activity provides a concrete context where those terms become meaningful.

Activities

Connections (continued)	Notes
13. Ask participants to flip the paper again. They should have four flaps on the top side (fig. 7). Ask them to fold one corner to the center (fig. 8). Then to ask them to fold the other three corners (fig 9).	
14. Ask participants to flip the paper again. Ask them to open two of the squares, opposite to each other, to form the two smokestacks (fig. 10). You may have to demonstrate how it is done table by table.	Some participants may need special guidance at this point.
15. As they fold the ship to put the smokestacks side by side, push out the prow and the stern to finish the ship (fig. 11). Again, you may need to demonstrate table by table.	
16. Another aspect that the instructor may want to emphasize as participants fold the ship is to look at the symmetries of the figures obtained in each step. The original square has rotational symmetry and two kinds of lines of mirror symmetry: the middle lines are one kind and the diagonals are another kind. Participants can fold the figures to convince themselves whether a line is indeed a line of symmetry or not.	
17. Participants can also see the symmetry of the final ship. The ship has two kinds of bilateral symmetry. Participants can imagine a mirror going prow to stern, another mirror perpendicular to this one going from the port side to the starboard. For participants not familiar with the maritime terminology, the planes of mirror symmetry can be described in terms of front-end and right-left.	

Closure (5 minutes)	Notes
Handouts: • BLMs 10.1-2: Van hiele Levels 1. The handout provides a brief overview of the Van Hiele levels of development in geometrical thinking. The instructor can present the levels on the overhead projector and discuss them. Participants reflect on the session. They can contrast ways to learn geometry using concrete representations of geometrical shapes with more abstract approaches and relate the activities to different levels of development of geometrical thinking.	

Activities

Closure (5 minutes)	Notes
2. The purpose is not for parents to memorize the different stages of the Van Hiele model. Rather, what is important for them is to realize that different children may be at different stages in their development of geometrical thinking. In the beginning, children will focus only on the shape as a whole, and even though they may be able to repeat words like "sides", they may not know what adults mean by that. Parents need to realize the importance of providing students with the opportunity to grow from one stage to the next. Just because children grow physically, it does not mean that automatically they will evolve in their geometrical thinking. Many people reach adulthood still thinking at the first level. This is clearly the case when working with parents. For several of them a square that is tilted was a diamond. Only after the corresponding activity were they comfortable calling it a square. Other parents remained at level 2 for most of the course, resorting to empirical verification, rather than trying to make short chains of deductive thinking. It is important therefore that the activity would make sense at that level too, and at the same time would be appropriate for developing more advanced thinking. 3. Many parents did not have the opportunity in school to learn geometry at a level appropriate for their own development of geometrical thinking. It is quite possible that many parents will be at the first level when the course starts. The instructor needs to be able to provide activities that make sense to all parents, independently of their level of development, and that at the same time offer the opportunity to grow from one stage to the next. 4. Hopefully by the end of the course most parents will have moved to the second level, and feel confident offering empirical evidence to convince themselves and others. It would also be very desirable that a considerable amount of the parents are at level 3 by the end.	This is not an easy task in the short duration of one course. However this was indeed the case observed with some parents. Towards the end of the course several of them, systematically tried to provide convincing arguments using logical thinking rather than just measuring. It is beyond this course to try to reach level 4.
Take Home Activities	
Distribute the following handouts for the participants to take home: • BLMs 11.1-2: Tangram Puzzle Solutions	
Preparation for the Next Session	
Collect name cards for use in the next sessions.	