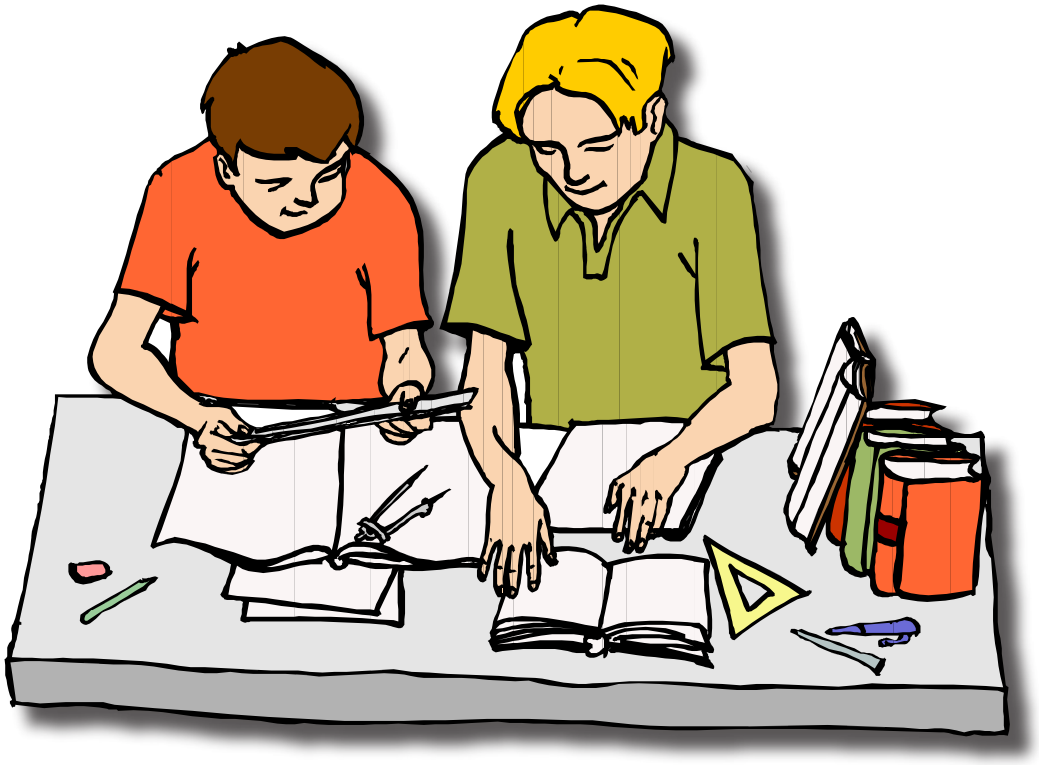
Problem Solving



Logic and Reasoning

Outcomes

- To explore skills employers desire in employees.
- To explore how these skills are reflected in how students are learning mathematics.
- To explore multiple strategies for the same problem to understand how differently people think about the same problem.

Overview

The main ideas of this workshop are working as a team to solve a problem and sharing strategies. In order to establish the importance of these skills, participants work on the activity named Fortune 500. The idea came from an article from Fortune 500. In it, major companies were asked to rank qualities that they desire in prospective employees. Teamwork, problem solving, interpersonal skills and communication skills were ranked as the top skills desired by employers.

After discussing the qualities from the Fortune 500 activities, participants solve a problem as a team. There are three options for the problem. The first should be used for workshops with parents and children in kindergarten, first and second grades. It is a combination of two questions about bikes, trikes, and wheels. The second is a pizza problem: "How many pizzas do the people at your school eat in a year?" This question is used with families in the workshop. The third question should be used when only parents are in the workshop, as it is not accessible to children. It was used in an interview for a major high-tech company: "How many gas stations are there in Tucson?" There are some thoughts on these problems in the mathematical background.

There is ample time in the session for teams to make posters and share the strategies and assumptions that they made when doing the problem. Sometimes there are unexpected connections between the teams and how they approached the problem. It is stimulating for the participants to see these connections. The richness in this activity are the many approaches to the problem.

Connections are made between the problem-solving experience, the national standards, and the skills from the Fortune 500 activity.

The next section of the session is important for making connections to the district's curriculum. It should be supplemented with examples and explanations of how your district encourages cooperative learning and problem solving.

The session ends with prizes for the estimations and the district evaluation or a final reflection.

The parent reference, Figure This!, must be ordered with ample time if it is to be used in Part 6. Ordering information can be found on **BLM 51: Resources**.

It is important to work the problems ahead of time in order to be aware of the challenges that they offer for participants. The workshop suggests using the problem, How many fingers are in this room?, as a mode. Organize this problem in a poster. The lettering in the poster needs to be large enough for all to see (modeling how big they will need to make their writing on the poster). The poster should state the problem, and then have a section for assumptions. Since the poster will be made ahead of time, you will not know how many people will be attending the session. Assumptions would include how many fingers each person has, and how many people might be attending the workshop. Your assumption will be wrong, but it will be based on logical thinking. You can explain that thinking during the workshop. Write "my method", then do the mathematics of the problem: 10 x 18 (or however many people you assume will be in the workshop). Finish the problem by listing the answer under a section called "my solution".

Mathematics Background

The focus of this module is problem solving and teamwork.

Connections to Principles and Standards for School Mathematics
The National Council of Teachers of Mathematics includes problem solving as a standard to be taught in all instructional programs, starting at the prekindergarten level. Problem solving should enable all students to:

a) Build new mathematical knowledge through problem solving.
b) Solve problems that arise in mathematics and in other contexts.
c) Apply and adapt a variety of appropriate strategies to solve problems.
d) Monitor and reflect on the process of mathematical problem solving.

Posing problems comes naturally to young children: They wonder about things like how long it would take to count to a million, or how many soda cans would it take to fill the school building. Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds. Teachers play an important role in the development of students' problem-solving dispositions by creating and maintaining classroom environments in which students are encouraged to explore, take risks, and share failures.

Also, the standards of reasoning and proof (developing and evaluating mathematical arguments), communications (communicating mathematical thinking coherently and clearly to peers, teachers and others), and connections (opportunities for students to experience mathematics in context, applied to their daily lives) are all integral parts of this module.

Team Work
In order to make connections between the NCTM Principles and Standards and the work world, the participants work on the activity named Fortune 500. The idea came from an article from Fortune 500. In it, major companies were asked to rank qualities that they look for in prospective employees. Teamwork, problem solving, interpersonal skills and communication skills are ranked as the top skills desired by employers.

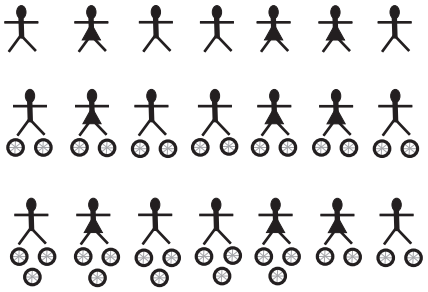
Assumptions
In each of the problems there is ambiguity. Ambiguity leads to open discussions about the thought process that was involved in solving the problem. One of the processes is assumptions. Assumptions are the estimations of facts that are needed in order to solve the problem. With the pizza problem, participants need to make assumptions about how much pizza each individual eats in a year and how many people are in the school. Since there is no information on these exact numbers, participants have the opportunity to use logical thinking to estimate this information, and then discuss how they decided on the different assumptions. This leads to rich mathematical dialogue.

The Problems
Problem 1A: Bikes, Trikes and Wheels
The more commonly recognized methods for solving problems include: Look for a pattern, construct a table, make an organized list, act it out, draw a picture, guess, check and revise, work backwards, write an equation, solve a simpler problem, then look for patterns, make a model, or use logical reasoning. We have seen this problem solved in a variety of ways from using pictures to algebra. A fun extension to the problem is to have each team name their strategy. Sometimes a participant solves it very quickly. Challenge that participant to solve it in a different way. And then even in another way if time permits.

Mathematics Background continued

Examples of some strategies to the problem stated below:
There are seven children riding bicycles or tricycles in the park. Joe counted 19 wheels. How many tricycles are they riding?

1. Drawing pictures and sharing thinking:



I know there are seven children and 19 wheels.

I gave each child 2 wheels.
I used 14 wheels, I have 5 left.

I gave 5 children another wheel.
So, there are 5 trikes.

2. Manipulatives:
The problem could be done with manipulatives in a similar fashion to the drawing above.

3. Guess, check, and revise:
Guess and check and revise is often looked down upon, but there is a lot of mathematical thinking going on when students get good at guess and check. They are developing their number sense: how operations affect numbers and how changes in operations change the results.

4. Looking for a pattern and using a table:

Bikes	Trikes	Wheels
7	0	14
6	1	15
5	2	16
4	3	17
3	4	18
2	5	19
1	6	20
0	7	21

When participants try to prove that there is only one solution, this table is a great way to prove that all the possibilities have been considered.

5. Algebra:

B represents the number of bikes and T represents the number of trikes
2B + 3T = 19 (2 wheels for each bike + 2 wheels for each trike = 19)
B + T = 7 (Number of bikes plus number of trikes equals 7)

$2B + 3T = 19$
$(B + T = 7) \times 2 = 2B + 2T = 14$

$2B + 3T = 19$
$-\ (2B + 2T = 14)$
$T = 5$

Problem 1B: More Bike, Trikes and Wheels
There are several children riding bicycles or tricycles in the park. Joe counted 21 wheels. How many children are in the park, and how many tricycles are they riding? Is there more than one solution? This problem is similar to Bikes, Trikes, and Wheels. This problem is more open than the other problem. It has no specified number of children, so there are several possible answers.

Mathematics Background continued

Below is a table that shows the possibilities.

Children on Bikes	Children on Trikes	Wheels	
0	6	18	Too few wheels, need more vehicles
0	7	21	Yes!
1	6	20	
2	6	22	
3	5	21	Yes!
4	4	20	
5	4	22	
6	3	21	Yes!
7	2	20	
8	2	22	
9	1	21	Yes!
10	0	20	
11	0	22	Too many wheels from here on out.
12	0	24	

So there is a solution when there are 7, 8, 9, or 10 vehicles. 6 vehicles are too few to have a solution because 6 trikes have a total of 18 wheels which is not enough wheels. Similarly, 11 vehicles have too many wheels.

Problem 2: That's a lot of Pizza!

How many pizzas are eaten by the people at your school in one year? There is ambiguity to this problem on purpose. The first question that will probably come up is: What size is the pizza? You can handle this by asking it before everyone starts and having everyone agree on the size, or you can let the size be one of the assumptions that the teams are making. Another question is: Who is included at the school? The intent was everyone. This makes the problem richer because some teams will include the principal, the janitor, the secretary and the bus drivers and some will not. Whatever assumptions they make, and whatever the answer, the richness of the question is in the diversity of strategies and the opportunity to share them. Since there is no "right" answer, each team's thinking can be celebrated and discussed openly. If a team finishes early, an extension question could be: How much money do the people of your school spend on pizza in a year?

Problem 3: Fill 'er up, please!

How many gas stations are there in our city? (This question can be changed to state or country.) Participants have found a variety of ways to solve this problem. Some have looked at main crossroads and assumed an average of 1, 1.5, or 2 gas stations at these crossroads. Others have estimated the number of cars and assumed that a gas station is needed for every 3,000 to 5,000 cars. In one workshop, a parent owned a station and took the number of gallons that he typically sold each week, decided how many cars that would be, and then estimated the number of cars needed for the population. From there he determined how many gas stations would be needed to service that number of cars. Surprisingly, the teams at this workshop were very close in their answers even though their approaches were quite different. It was quite a surprise to everyone. Whatever the method used or the answer, the richness of the question is in the diversity of strategies and the opportunity to share them. As the participants do their walk-about, they can pay attention to the team's method for organizing their thoughts and the assumptions that were made. If a team finishes early, an extension question could be: How much money do residents of your city spend on gas each week? Month? Or year?

Room Setup

- Desks or tables arranged in groups of 3-4
- Tables for sign-in, supplies, estimations, and snacks
- Overhead projector and screen
- Chart paper on easel
- Poster of the agenda

Materials

Facilitator	Transparencies
• Overhead projector • Overhead pens • Transparencies, blank • Chart paper • Chart markers • Colored markers • Masking tape • Timer (optional) • Estimation questions (prepared by facilitator) • Inexpensive prizes	*BLM 1: Welcome* *BLM 45: Fortune Magazine Cover* *BLM 47: Fortune 500 Companies* *BLM 48: Problem Solving* *BLM 49: NCTM Problem Solving Standard* *BLM 50: Assumption* *BLM 51: Bikes, Trikes, and Wheels* *BLM 52: More Bikes, Trikes, and Wheels* *BLM 53: That's a Lot of Pizza* *BLM 54: Fill 'er Up, Please!* *BLM 55: Poster Instructions* *BLM 56: Walk About Instructions*
Participant	**Handouts**
Individuals • Paper • Pencil • Calculators • Post-it notes • Envelopes with Puzzle for Home (prepared by facilitator) • Reflection **Groups** • Colored markers • Chart paper	**One per participant for class** *BLM 46: Preparing Skill Strips* **(copy, cut and prepare)** *BLM 47: Fortune 500 Companies* *BLM 49: NCTM Problem Solving Standard* **One per participant for home (make packet for easy distribution)** *BLM 57: Instructions for Puzzle for Home* *BLM 58: Puzzle for Home* **(copy on cardstock, cut and prepare)** *BLM 59: Resources*

Timing

2 hours

Preparation and Timing (2 hours)

Part 1: Getting Started (10 minutes)

Display transparency from workshop one:
BLM 1: Welcome

Part 2: Fortune 500 (30 minutes)

Make transparency of:
BLM 45: Fortune Magazine Cover
BLM 47: Fortune 500 Companies
Make copies for each participant:
BLM 46: Preparing Skill Strips **(copy, cut and prepare)**
BLM 47: Fortune 500 Companies

Part 3: Problem Solving (40 minutes)

Make transparency of:
BLM 48: Problem Solving
BLM 49: NCTM Problem Solving Standard
BLM 50: Assumption
BLM 55: Poster Instructions
Choose appropriate question for workshop and make transparency of:
BLM 51: Bikes, Trikes, and Wheels
BLM 52: More Bikes, Trikes, and Wheels ***or***
BLM 53: That's a Lot of Pizza ***or***
BLM 54: Fill 'er Up, Please!
Make copies for each participant:
BLM 49: NCTM Problem Solving Standard

Part 4: Walk About and Discussion (30 minutes)

Make transparency of:
BLM 56: Walk About Instructions

Part 5: Connections (5 minutes)

No handouts or transparencies

Part 6: Take Home Applications (5 minutes)

Make copies and prepare a take home packet for each participant:
BLM 57: Instructions for Puzzle for Home
BLM 58: Puzzle for Home **(copy on cardstock, cut and prepare)**
BLM 59: Resources

Part 7: Closing (5 minutes)

No handouts or transparencies
3-4 Inexpensive prizes for Estimation Question winners
Reflection / evaluations *(provided by the evaluation team)*

Facilitator Resources

Articles

Mikusa M, *Problem Solving is More Than Solving Problems,* Mathematics Teaching in the Middle School, September 1998, P. 20

Van Zoest L , and Enyart A , *Discourse, Of Course; Encouraging Genuine Mathematical Conversations,* Mathematics Teaching in the Middle School, November-December 1998, P. 150.

Fortune Magazine Cover, Fortune, Vol. 147, No. 1, January 20, 2003, www.fortune.com/

The Top 500 Companies, Fortune, www.fortune.com/fortune/fortune 500

Lost in Mathland — California's Controversial New Curriculum, Sacramento News & Review , Vol. 9, No. 2, April 27, 1997, www.newsreview.com

Books

Standards 2000 Project, *Principles and Standards for School Mathematics*, The National Council of Teachers of Mathematics, Inc (NCTM), 2000, p. 116, ISBN 0-87353-480-8, www.nctm.org

Kagan, Dr. Spencer, Cooperative Learning, *Thinking Skills Structure,* Resources for Teachers, Inc., 1-800-Wee Co-op. 1994, P. 11:8, ISBN 1-879097-10-9, www.KaganCoopLearn.com

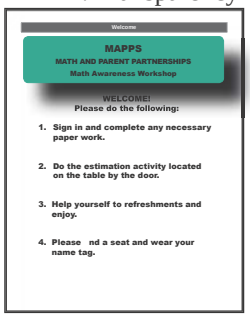
Internet Sites for Parents to Explore

Site addresses updated 9/21/03

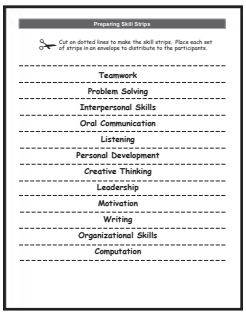
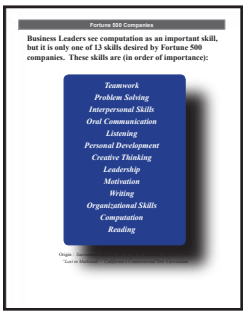
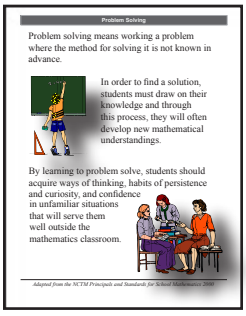
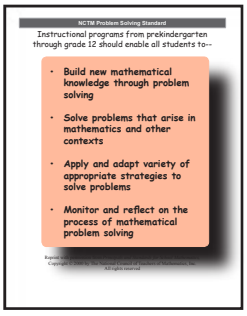
Figure This!: http://www.figurethis.org/

Math Forum: http://mathforum.org/

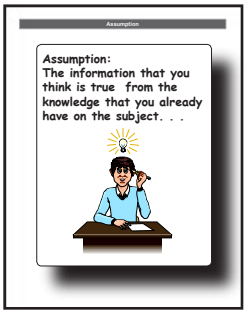
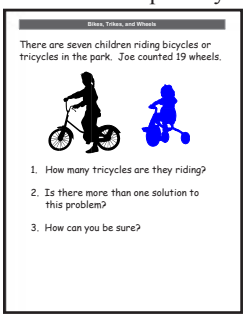
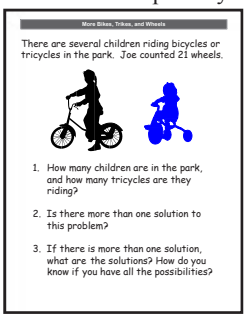
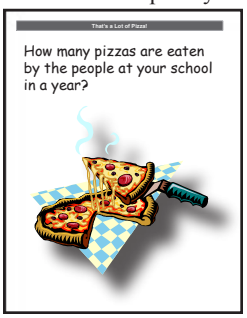
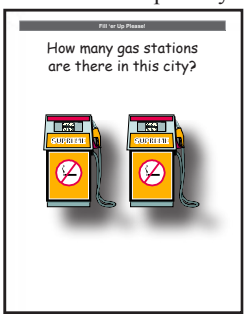
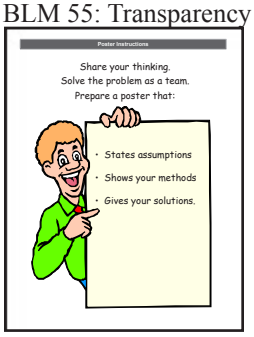
Activities

Preparation of Classroom	Notes
1. Set up a table with a sign-in sheet, name tags, and snacks. On another table set up estimation activities. Arrange desks or tables in groups of 4-6. 2. Display the transparency of **BLM 1: Welcome!**. 3. Prepare and display a poster with the agenda and purpose of the session. 4. Prepare a poster of the sample problem: • How many fingers are in this room? Follow the format that is given to the participants: • What is the problem? What are your assumptions? • What is your method? • What is your solution? For a futher discussion of this poster, see the overview.	BLM 1: Transparency  Welcome MAPPS MATH AND PARENT PARTNERSHIPS Math Awareness Workshop WELCOME! Please do the following: 1. Sign in and complete any necessary paper work. 2. Do the estimation activity located on the table by the door. 3. Help yourself to refreshments and enjoy. 4. Please nd a seat and wear your name tag.
Part 1: Getting Started (10 minutes) - parents and children	
Introductions 1. Introduce yourselves and then have the participants introduce themselves. 2. Briefly explain the MAPPS program. Have participants who are involved in the program share their experiences. 3. Give participants an overview of the session. Review the agenda and purpose of the session. When discussing the agenda, let the participants know the plan for including children in the session.	
Part 2: Fortune 500 (20 minutes)	
1. Set the stage by displaying the **BLM 45: Fortune Magazine Cover** transparency and asking the question: *We hear a lot about what skills our students need to have when they leave high school. Have you ever wondered what the top 500 American companies indicate are the critical skills necessary for success?* See Note A on next page.	BLM 45: Transparency  FORTUNE 100 BEST COMPANIES TO WORK FOR

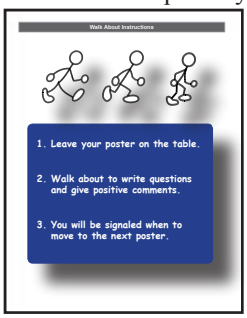
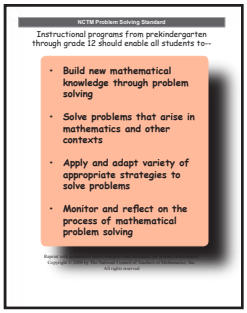
Activities

Part 2: Fortune 500 (continued)	Notes
2. Group ranking: a. Have participants sit in groups and distribute one envelope to each group with the 13 skill strips, that have been previously prepared according to the instructions on the **BLM 46: Preparing Skill Strips** handout for facilitator. b. Have individuals choose what they think would be the top three skills. c. Have groups reach consensus and place all thirteen skills in order from most important to least important. d. Visit groups while they are working to answer questions and observe. 3. Processing and review: a. Ask each table group for their top 3 ranking skills. b. Write the responses on a blank overhead transparency. c. Have the participants identify the skills ranked as most important by the entire group. d. Distribute **BLM 47: Fortune 500 Companies**, handout of the ranking by the Fortune 500 Companies and display the transparency. 4. Ask the following discussion questions: • *If these are skills people need in order to be effective in the workplace, then when and how should they be learned?* • *Which of these skills should affect how mathematics is taught?* Have the groups discuss the questions and then share their thoughts with the whole class.	**A. NOTE:** The top 500 companies are according to Fortune magazine. To see a list of the Fortune 500 companies go to website www.fortune.com. The critical skills necessary for success are from an article in the Sacramento News & Review , Vol. 9, No. 2, 4/27/97. BLM 46: Handout  Preparing Skill Strips Cut on dotted lines to make the skill strips. Place each set of strips in an envelope to distribute to the participants. Teamwork Problem Solving Interpersonal Skills Oral Communication Listening Personal Development Creative Thinking Leadership Motivation Writing Organizational Skills Computation BLM 47: Transparency  Fortune 500 Companies Business Leaders see computation as an important skill, but it is only one of 13 skills desired by Fortune 500 companies. These skills are (in order of importance): *Teamwork* *Problem Solving* *Interpersonal Skills* *Oral Communication* *Listening* *Personal Development* *Creative Thinking* *Leadership* *Motivation* *Writing* *Organizational Skills* *Computation* *Reading*
Part 3: Problem Solving (40 minutes)	
1. Tell participants that problem solving is a very important part of mathematics. Today, problem solving is used in order to discover new ideas as well as to apply mathematical knowledge to new situations. Display the transparency of **BLM 48: Problem Solving** to highlight main ideas of problem solving from the NCTM standards. Distribute the **BLM 49: NCTM Problem Solving Standard** handout. 2. Tell participants that they are about to be given a problem but before they are given it, they need to think about assumptions.	BLM 48: Transparency BLM 49: Transparency /Handout  Problem Solving Problem solving means working a problem where the method for solving it is not known in advance. In order to find a solution, students must draw on their knowledge and through this process, they will often develop new mathematical understandings. By learning to problem solve, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. *Adapted from the NCTM Principals and Standards for School Mathematics 2000*  NCTM Problem Solving Standard Instructional programs from prekindergarten through grade 12 should enable all students to-- • Build new mathematical knowledge through problem solving • Solve problems that arise in mathematics and other contexts • Apply and adapt variety of appropriate strategies to solve problems • Monitor and reflect on the process of mathematical problem solving

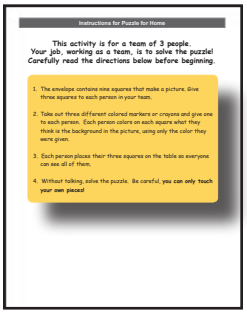
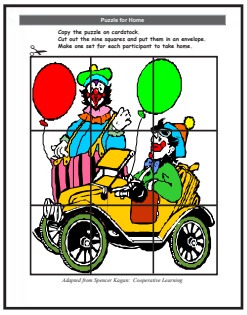
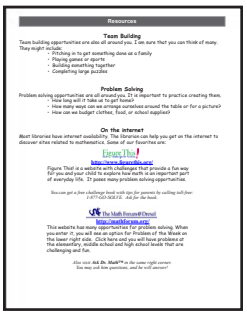
Activities

Part 3: Problem Solving (continued)	Notes
3. Display the **BLM 50: Assumptions** transparency, then discuss the use of assumptions in problem solving. Tell the participants that in order to solve a problem, they will have to make assumptions. Give an example of an assumption that is made on a problem: *How many fingers do we have in this room?* Share the poster that was made up ahead of time. Explain the logical thinking behind the assumptions. Point out that it was impossible to know exactly how many people would be present in the workshop and so this was the best that could be done with the information that you had. 4. Determine which of the following questions is appropriate for the session and use the corresponding transparency for **BLM 51 or 52** to present the question. a. For family workshops with K - 2nd grade students, use **BLM 51 and/or BLM 52:** **Bikes, Trikes and Wheels** *There were seven children riding bicycles and tricycles in the park. Joe counted 19 wheels. How many tricycles were they riding?* **More Bikes, Trikes and Wheels (optional)** *There were several children riding bicycles and tricycles in the park. Joe counted 21 wheels. How many children are in the park, and how many tricycles are they riding?* b. For family workshops with 3rd - 4th grade students, use **BLM 53: That's a lot of Pizza!:** *How many pizzas are eaten by the people at your school in a year?* c. For parent only workshops use **BLM 54: Fill 'er up, Please!:** *How many gas stations are there in Tucson?* (Replace Tucson with your city) 5. Have participants think about the question on their own for about five minutes and talk to their group about how they might answer this question. Let them know that there are calculators available for their use. 6. Display the transparency of **BLM 55: Poster Instructions** to create a team poster. Participants should work in groups of three or four to solve the problem and develop a poster that:	BLM 50: Transparency  BLM 51: Transparency  BLM 52: Transparency  BLM 53: Transpareny  BLM 54: Transparency  BLM 55: Transparency 

Activities

Part 3: Problem Solving (continued)	
a. States their assumptions. b. Shows their methods. c. Gives their solution. While the teams are working, be alert to what is happening, asking questions and directing the preparation of posters. For groups that finish early, an extension question for each problem is listed in the mathematics background. Give the groups a five or ten minutes warning to complete their posters.	
Part 4: Walk About and Discussion (30 minutes)	**Notes**
1. Display the transparency of **BLM 56: Walk About Instructions** for the instructions for a "walk about". Participants need to have post-it notes and pens or pencils to write comments on each other's posters as they walk about. 2. When the walk-about is complete, process it by asking the following questions: • *What mathematics did you do in this problem?* • *What did you notice in the walk about?* • *What surprised you?* The discussion should include: a. Mathematical processes. b. Similarities and differences in approaches to the problem.	BLM 56: Transparency 
Part 5: Connections (5 minutes)	
1. Review the ideas that came out of Fortune 500. Ask participants which of these skills were important in the activity of the session. 2. Revisit the standards on problem solving from the NTCM Standards by displaying the **BLM 49: NCTM Problem Solving Standards** transparency and leading a discussion on what occurred (in relation to the standards) while problem solving. These standards have been developed by thousands of math educators who have looked closely at the needs of today's students. Ask participants how these standards relate to the activity of the session.	BLM 49: Transparency /Handout 

Activities

Part 6: Take Home Applications (5 minutes)	Notes
3. Let participants know that students are or should be learning using these skills in their classrooms. They will see this influence as they attend additional MAPPS sessions and while visiting their children's classrooms. This will help them have a better understanding of how and what their children are learning. 4. Tell participants that this session's goal was to introduce them to: a. The importance of seeing problems solved using different strategies. b. The connection between what employers value and how problem solving can be used in mathematics. 5. Wrap up the session with an introduction to the handouts for parents. **BLM 57: Instructions for Puzzle for Home** **BLM 58: Puzzle for Home** **BLM 59: Resources** **BLMs 57-58** contains an enjoyable puzzle for working together with others. **BLM 59** is a handout of resources for problem solving ideas. You may want to show participants a copy of Figure This!. Computers with internet access are usually available at the local library, if not available at the school.	BLM 57: Handout  Instructions for Puzzle for Home This activity is for a team of 3 people. Your job, working as a team, is to solve the puzzle! Carefully read the directions below before beginning. 1. The envelope contains nine squares that make a picture. Give three squares to each person in your team. 2. Take out three different colored markers or crayons and give one to each person. Each person colors on each square what they think is the background in the picture, using only the color they were given. 3. Each person places their three squares on the table so everyone can see all of them. 4. Without talking, solve the puzzle. Be careful, you can only touch your own pieces! BLM 58: Handout  Puzzle for Home Copy the puzzle on cardstock. Cut out the nine squares and put them in an envelope. Make one set for each participant to take home. Adapted from Spencer Kagan Cooperative Learning BLM 59: Handout  Resources Team Building Team building opportunities are also all around you. I am sure that you can think of many. They might include: • Pitching in to get something done as a family • Playing games or sports • Building something together • Completing large puzzles Problem Solving Problem solving opportunities are all around you. It is important to practice creating them. • How long will it take at home to… • How many ways can we arrange ourselves around the table or for a picture? • How can we budget clothes, food, or school supplies? On the Internet Most libraries have internet availability. One of our librarians can help you get on the internet to discover sites related to mathematics. One of our favorites are: Figure This! http://www.figurethis.org/ Figure This! is a website with challenges that provide a fun way for you and your child to explore how math is an important part of everyday life. It poses many problem solving opportunities. You can get a free challenge book with tips for parents by calling toll-free: 1-877-GO-MATH. Ask for the book. The Math Forum@Drexel http://mathforum.org/ This website has many opportunities for problem solving. When you enter it, you will find the Problem of the Week on the lower right side. Click here and you will have problems at the elementary, middle school and high school levels that are challenging and fun. Also visit Ask Dr. Math™ in the same right corner. You may ask him questions, and he will answer!
Part 7: Closing (5 minutes)	
1. If your district does not have an evaluation form to use, have them answer one of the following questions: • *Was the room quiet or did people talk to each other to learn mathematics?* • *Was the conversation "cheating"?* • *What tools did you use?* ` • *Did the facilitators tell the answers?* The discussion can center around the idea that noisy classrooms are not out of control classrooms, and that discussing problems can be very enlightening for students. 2. Distribute any prizes from estimations or drawings.	