

SESSION SEVEN

ESTIMATING AND WRITING LARGE NUMBERS

Outcomes

- To experience the magnitude of large numbers.
- To provide opportunities to estimate using scientific notation.
- To engage in mathematical reasoning as it relates to large numbers.

Overview

The seventh session of Thinking About Numbers focuses on large numbers. Participants explore the largest numbers that they have seen, and then compare that number to a trillion. The activities are designed to build on participants' experiences with large numbers and to explore the use of scientific notation. The session ends with investigations into the principles behind large numbers.

Time

- 10-15 minutes** The first part of the session allows participants to discuss their take home activities.
- 20-30 minutes** The second activity of the session builds on the experiences from the last session. Participants are asked to think of a shorthand method for writing very large numbers. Their shorthand methods are connected to the powers of ten. A worksheet is distributed that helps participants establish the relationship between the places in a number and the powers of ten.
- 20-30 minutes** Participants learn to write and do operations in scientific notation.
- 20-30 minutes** The knowledge of scientific notation is used to estimate large numbers like the distance from the Earth to the Sun.
- 15-20 minutes** Participants investigate mathematical reasoning by exploring true/false statements.
- 15-20 minutes** In the closing activity, participants share their ideas about the advantages and disadvantages of using scientific notation.

Materials

Facilitator	Transparencies (English & Spanish)
<ul style="list-style-type: none"> • Overhead calculator 	<i>BLM 38.1-2: Powers of Ten</i> <i>BLM 41: Principles Behind Large Numbers</i>
Participant	Handouts (English & Spanish)
<ul style="list-style-type: none"> • Calculators • Rulers • Pennies (about 50 per group) 	<p>Two per participant for class and home <i>BLM 38.1-2: Powers of Ten</i> <i>BLM 39: Scientific Notation</i> <i>BLM 40: Estimating with Scientific Notation</i> <i>BLM 41: Principles Behind Large Numbers</i></p> <p>One per participant for home <i>BLM 42.1-2: Bringing Mathematics Home 7</i> <i>BLM 43: Estimating and Writing Large Numbers</i></p>

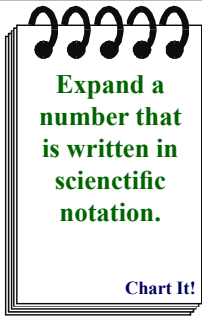
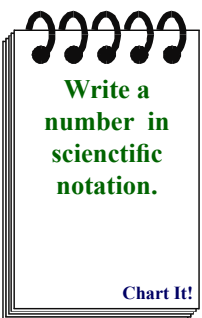
Activities

Preparation of Classroom	Notes
<ol style="list-style-type: none"> 1. Set up the Chart It! 2. Place the name cards from last class near the front of the room where participants can easily find them. 3. Around the room, place several numbers that are extremely large, each having a one in the largest place, and followed by 24, 26, 28, and 30 zeros. Place them so that they are not lined up and are not near to each other. 4. Check the calculators that you will be using ahead of time to see how they display scientific notation. 	
Discussion of Homework (10-15 minutes)	
<ol style="list-style-type: none"> 1. Have participants discuss their experience when playing the game with their children. <ul style="list-style-type: none"> • What mathematical thinking was brought out by playing the game? • What surprised you when playing the game? <p>Take a few minutes for them to share with their groups and then ask for a few volunteers to share with the whole class.</p> 2. Answer any questions that the participants have about Exponential Notation. 	
Thinking About Large Numbers (20-30 minutes)	
<p>Introduction</p> <ol style="list-style-type: none"> 1. The largest number: <ul style="list-style-type: none"> • Ask participants to think about the largest number that they have ever seen. • Have them write it in numbers or words. • Have the participants share their numbers. • Have them compare their number to one trillion. 2. One million: <ul style="list-style-type: none"> • Ask participants what they think of when they think of a million. Is there an example from real life that they think must be close to one million in number? • Tell them that one million pennies sound like a lot of money. What is the exact value? 	<p>The idea of gazillion might be mentioned. There is no gazillion designation in our numbering system. Others might bring up a googol. A googol is one followed by one hundred zeros.</p> <p>Examples of real life situations that involve one million:</p> <ul style="list-style-type: none"> • The cost of some houses • Population of some cities • Sales of top 10 CD's • # of people seeing a popular movie <p>One million pennies is worth: \$10,000.</p>

Activities

Thinking About Large Numbers (continued)	Notes
<ul style="list-style-type: none"> • Tell them that one million pennies sound like a lot of money. What is the exact value? • Then ask them how long they think that it would take to count to one million if we counted one number every second. How long would it take for one million seconds to pass? <ul style="list-style-type: none"> - Have them make a guess and write it down on a sheet of paper. - Have them take some time at the table to talk to others and put some reasoning into their guesses. - Have the groups compute the amount of time that would pass using their calculators. <p>Introducing powers of ten</p> <ol style="list-style-type: none"> 1. Explain to the participants that the Greek mathematicians had difficulty expressing larger numbers. How confusing it would be to read numbers that are very large like we have posted around the room! <ul style="list-style-type: none"> • Ask participants to work with a partner and figure out which of the numbers posted around the room is the largest number. • Have them create a shorthand way to represent these numbers. • After discussing the methods, have the participants put the smallest of the numbers in their calculators and see how the calculator deals with such large numbers. • Ask them what the calculator is doing. How does this compare with the methods that they created? 2. Hand out the worksheet Powers of Ten. <ul style="list-style-type: none"> • Have participants work on the worksheet • Circulate to answer questions. • Display a transparency of Powers of Ten on the overhead and have participants share their answers. 3. Introduce the concept of scientific notation by explaining that scientists found the powers of ten to be a great shortcut. Scientists have found it helpful to write large numbers in the form of 1×10^{21}. We call this form scientific notation. 	<p>There are 86,400 seconds in each day ($60 \times 60 \times 24$)</p> <p>One million seconds are 11.57 days.</p> <p>This simplifies to 11 days, 13 hours, 46 minutes and 40 seconds and is approximated as a little less than 12 days.</p> <div data-bbox="1117 520 1318 835" data-label="Image"> </div> <p>To compute it, we need: 60 seconds times 60 minutes (# of seconds in an hour) times 24 hours (number of seconds in a day).</p> <p>*Check the calculators that you will be using ahead of time to see how they display scientific notation.</p> <ul style="list-style-type: none"> • A graphing calculator will display the number 1 with 24 zeros as 1 E 24. • A scientific calculator will display the number as 1.0×10^{24} or $1_{x10} 24$. <p>Some calculators will not write 24 zeros. When this happens, have the participants multiply one million by one million in order to see the scientific notation used by the calculator.</p>

Activities

Scientific Notation (20-30 minutes)	Notes
<p>Scientists, as you can imagine, deal with very large numbers regularly. Therefore it was necessary for them to develop a shorthand method to use as a standard for all scientists. Their method is explained below.</p> <p>1. Hand out Scientific Notation. Let participants know that some of these problems may be puzzling. They are designed to create curiosity about further skills in scientific notation. The participants are not expected to know all the answers. Encourage them to be curious.</p> <ul style="list-style-type: none"> Have participants work on the first two problems only. Lead a discussion about how the exponent in the calculator does not refer to the number of zeros, but to the power of 10 that is used for expanding the number. Say: <i>Your calculator may read: 2.7 13. However, when you count the zeros involved in the problem, there are only 12.</i> Then ask: <i>Why does your calculator say 13?</i> <i>What is it doing?</i> <p>2. It is convenient to think of this as the number of places to move the decimal place to the right.</p> <ul style="list-style-type: none"> Illustrate this connection by showing 2.7×10^2. Show moving the decimal point 2 places to the right, and also show that 10^2 is 100 and that 2.7×100 is 270. 	<div data-bbox="1084 254 1284 569">  <p>Expand a number that is written in scientific notation. Chart It!</p> </div> <p>To expand a number that is written in scientific notation, move the decimal place to the right.</p> <div data-bbox="1084 695 1284 1010">  <p>Write a number in scientific notation. Chart It!</p> </div> <p>To write a number in scientific notation, place the decimal behind the largest place value and count the number of places to the right of the new decimal point then write $\times 10$ to that power.</p>
Applications of Scientific Notation (20-30 minutes)	
<p>1. The importance of the activities on the application sheet is the opportunity to experience the ease of multiplying and estimating when using scientific notation.</p> <p>2. Hand out Estimating with Scientific Notation.</p> <ul style="list-style-type: none"> Have participants work on the problems. Circulate the room, encouraging participants to use scientific notation to estimate the answers. Have participants share their reasoning with the class. <ul style="list-style-type: none"> Have them share how they rounded off each number. Have them show the estimation that they made using scientific notation. 	<p>These activities use only multiplication. The idea of estimating should be emphasized. The exact answer should be used only to check on the accuracy of the estimate.</p> <p>Question 1: Estimate with scientific notation: Round 186,000 to 200,000 and 31,536,000 to 30,000,000. Putting both in scientific notation $2 \times 10^5 \times 3 \times 10^7$. When using exact figures on the calculator to check the estimate, the answer is 5.865696 12. This rounds off nicely to 6.0 12.</p>

Activities

Applications of Scientific Notation (continued)	Notes
<ul style="list-style-type: none"> - Spend more time discussing the estimation and underplay the exact answer. - Explain that scientific notation is also used for very exact computations, but that today the emphasis was on estimation. 	<p>Question 2: $186,000 \times 500$ Estimate it as: $2 \times 10^5 * 5 \times 10^2 = 10 \times 10^7 = 100,000,000$ The exact answer is 93,000,000.</p> <p>Question 3: Hand out pennies and rules: One foot is approximately 240 pennies.</p>
Principles Behind Large Numbers (15-20 minutes)	
<p>Hand out Principles Behind Large Numbers.</p> <ul style="list-style-type: none"> • Give the participants time to read the questions to see if they understand them. • Answer any questions that come up • Give participants time to complete the three problems. • Have different groups share their reasoning for each of the problems. 	
Closure (10 minutes)	
<p>Participants reflect on the session. Ask: <i>What would be the advantages and disadvantages of using scientific notation?</i></p> <p>Ask a few volunteers to share their reflections</p>	
Take Home Activities (5 minutes)	
<ol style="list-style-type: none"> 1. There are five handouts for participants to take home: <ul style="list-style-type: none"> • Bringing Mathematics Home 7 • Estimating and Writing Large Numbers • Powers of Ten • Scientific Notation • Estimating with Scientific Notation • Principles Behind Large Numbers 2. Have participants look through the packet of materials as you explain them. The object of the take home activities is for them to practice with their children. Therefore, they need fresh copies of the session's activities. 3. Let participants know that they should be ready to share their experiences at the next session. 	<p>Many facilitators find it pleasant to have a pot luck on the last session. If you want a pot luck then organize it.</p>

Activities

Preparation for the Next Session	
<div>1. Optional: Prepare certificates for the next session.</div> <div>2. Optional: Check your district for an evaluation process with class.</div> <div>3. Collect name cards for use in the next sessions.</div> <div>4. Save the Chart It! and bring it to the next class. If desired, you may have the log typed and distributed to participants at the next class.</div>	