

## SESSION THREE

### DIVISION OF WHOLE NUMBERS

#### Outcomes

- To explore different models of division
- To make connections between division and subtraction
- To investigate ways of expressing remainders
- To use mathematical reasoning

#### Overview

The theme of the third session is understanding division. Division is more complicated than multiplication, and many children have difficulty doing long division. Though they memorize the procedure, they do not understand why it works. The answer to division seems strange because it can be represented in many different ways: a quotient with a decimal, a fraction, or a remainder. This session tries to clarify the procedures of long division with sharing money.

#### Time

- 10-15 minutes** The session starts with a review of the take home activities from the last session.
- 30-40 minutes** Next participants work with long division. They start with doing a long division problem. They examine the different steps to doing the problem. Then they do the same problem as a hands-on activity, sharing money. This sharing is processed and recorded. Connections are made between distributing money and long division.
- 30-40 minutes** The next activity is a look at the three different representations for remainders: as a decimal, fraction, or a whole number remainder. Participants are given different situations and asked which form is the most helpful for each particular situation.
- 15-20 minutes** After the explorations with division, participants investigate conjectures and patterns involving division.
- 15-20 minutes** In the closing activity, participants think about how they were taught division, what problems their children might have with division, and how they might help their children understand the process of long division.

#### Materials

Facilitator	Transparencies (English & Spanish)
<ul style="list-style-type: none"> <li>• Transparent dollars dimes and pennies</li> <li>• Overhead calculator</li> </ul>	<i>BLM 14: Sharing Money Equally</i>
Participant	Handouts (English & Spanish)
<ul style="list-style-type: none"> <li>• Play money: 20 one-dollar bills, 20 ten-dollar bills and 10 hundred-dollar bills for each group of 4</li> <li>• Play coins: 50 dimes and 30 pennies for each group of 4</li> <li>• Calculators that can divide and give remainders (even better if it will also show the remainder as a fraction)</li> </ul>	<p><b>Two per participant for class and home</b>  <i>BLM 14: Sharing Money Equally</i>  <i>BLM 15: Applications of Division</i>  <i>BLM 16: Division Principles</i></p> <p><b>Make four copies of BLM 17 and two copies of BLM 18 per participant</b>  <i>BLM 17-18: Money Templates</i></p> <p><b>One per participant for home</b>  <i>BLM 19: Bringing Mathematics Home 3</i>  <i>BLM 20: Division with Whole Numbers</i></p>

## Activities

Preparation of Classroom	Notes
<ol style="list-style-type: none"> <li>1. Since there are several take home activities in this session, it would help to make a packet for participants ahead of time.</li> <li>2. Set up the <b>Chart It!</b></li> <li>3. Place the name cards from last class near the front of the room where participants can easily find them.</li> <li>4. Have a supply of base ten blocks on the tables.</li> </ol>	<p>As you use the Chart It!, take the papers off and display them. The long division and the shortened version of repeated subtraction should be posted side by side.</p>
Discussion of Homework (10-15 minutes)	
<ol style="list-style-type: none"> <li>1. Open the session by asking participants to share one of the funnier stories that their family created this week for multiplication (<b>Bringing Mathematics Home 2</b>, problem #2).</li> <li>2. Invite participants to share with a partner any other experiences working with their children on <b>Bringing Mathematics Home 2</b>.</li> <li>3. Next, ask them to share their solutions to <b>Multiplication with Whole Numbers</b> with the people at their table by discussing these questions: <ul style="list-style-type: none"> <li>• Do you all agree on your answers?</li> <li>• What was your conclusion to problem #2 and why?</li> <li>• How did you make sense of problem #3?</li> </ul> <p>After everyone has had the opportunity to share with their group, you may ask one or two participants to share with the class</p> </li> </ol>	<p>It is important to set a relaxed and congenial atmosphere in the class. Doing a small icebreaker will help to develop the feeling of camaraderie. It also gives participants a chance to practice expressing their thoughts.</p>
Division by Successive Subtraction (30-40 minutes)	
<p>In this activity, participants will do a problem with paper and pencil, and then act it out with money. Afterwards, ways that this situation can be recorded are modeled.</p> <p><b>Connections to prior knowledge</b></p> <ol style="list-style-type: none"> <li>1. Ask participants to do the following long division problem: <math display="block">4 \overline{)975}</math> </li> <li>2. Have them explain to their group what they did and in particular, talk about all the marks that they put on the page. Circulate to see how participants are explaining their method</li> </ol>	

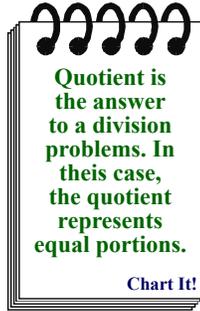
**Activities**

Division by Successive Subtraction (continued)	Notes
<p>3. Process the problem.</p> <ul style="list-style-type: none"> <li>Pick someone to model the problem on the overhead. Be careful to pick someone who has done the problem with a remainder, not with a decimal or a fraction.</li> <li>Discuss it.</li> <li>Make any necessary corrections.</li> <li>Record it on chart paper so that you can refer back to it to make connections.</li> </ul> <p>4. Ask:</p> <ul style="list-style-type: none"> <li><i>How would you explain this process to your children?</i></li> <li><i>Why do you divide 4 into 9 first and ignore the other numbers?</i></li> <li><i>What does the answer to the division problem mean?</i></li> </ul> <p><b>Modeling division</b></p> <p>1. Have participants act out division with play money.</p> <ul style="list-style-type: none"> <li>Have participants work in groups of 4.</li> <li>Have them get out \$975</li> <li>Have them split it equally among 4 people.</li> </ul> <p>2. Hand out <b>Sharing Money Equally</b> for participants to record the steps they took in passing out their money.</p> <p>3. Ask a participant to demonstrate how they shared the money among their group members.</p> <p>4. Demonstrate the scaffolding (repeated subtraction) method and at the same time, record how it looks on the division problem. (Refer to recording division answer key.) Afterwards, show it in shortened form.</p> <p>5. Ask:</p> <ul style="list-style-type: none"> <li><i>What is the connection between subtraction and division?</i></li> <li><i>What operation is multiplication related to?</i></li> <li><i>What do the scaffolding method and long division have in common?</i></li> </ul> <p>If the participants have not made the connection between the activity with the money and the place values of the long division, ask some of the following questions:</p>	<div data-bbox="1149 331 1349 642" style="border: 1px solid black; padding: 5px; text-align: center;">  <p><b>Record the long division problems with the remainder of 3</b></p> <p><small>Chart It!</small></p> </div> <p>Parents most likely will not be able to answer these questions. This is part of the discovery of the evening, so do not try to explain it at this time.</p> <p>It is important that the groups be exactly 4 so that the original problem is modeled with the money. If a group has 5 members, have one member act as the banker.</p> <p>These two methods should be processed at the same time so that the connections can be made between distributing money and the long division problem.</p> <div data-bbox="1149 1325 1349 1635" style="border: 1px solid black; padding: 5px; text-align: center;">  <p><b>9 represents the 9 one-hundred dollar bills (see below for other examples)</b></p> <p><small>Chart It!</small></p> </div> <p>1 represents the 1 one-hundred dolard bill we had left over.                      17 represents the 17 ten-dollar bills we had after we exchanged the hundred for 10 ten-dollar bills.</p>

**Activities**

Division by Successive Subtraction (continued)	Notes
<ul style="list-style-type: none"> <li>• <i>When you were distributing the money, what did you start with?</i> (nine one hundred dollar bills)</li> <li>• <i>Where do you see that 9 on the long division problem?</i></li> </ul> <p>Lead them into the discussion of how each number relates to the money.</p>	
Different Faces of Division (30-40 minutes)	Notes
<p><b>Remainder as a Whole Number</b> Remind participants that they had 3 dollars left over and that this is commonly written as R3 to represent a remainder of three.</p> <p><b>Remainder as a Decimal</b></p> <ol style="list-style-type: none"> <li>1. <b>Ask:</b> <i>How would your group split up the \$3 that you have left?</i> <ul style="list-style-type: none"> <li>• Have participants share their ideas and record them so that you can come back to them later.</li> <li>• Hand out dimes and pennies for each group.</li> <li>• Ask them to use these for sharing their \$3.</li> <li>• At the overhead have a volunteer demonstrate how they shared their money.</li> </ul> </li> <li>2. <b>Process their division of the \$3</b> <ul style="list-style-type: none"> <li>• Demonstrate how it looks to divide 3 by 4. Show one additional decimal place at a time, asking what it represents.</li> <li>• Ask them what their total share of the money is.</li> </ul> </li> <li>3. <b>Say:</b> <i>Let's record the other way to write our answer.</i></li> <li>4. <b>Have the participants try a problem on their own.</b> <ul style="list-style-type: none"> <li>• Have them divide 22 by 5.</li> <li>• Circulate to check for understanding.</li> <li>• Have someone share their answer and how they computed it.</li> </ul> </li> </ol> <p><b>Remainder as a Fraction</b> Have the participants use their calculators to discover the fraction representation of the remainder.</p> <ul style="list-style-type: none"> <li>• Have the participants take out their calculators and divide 975 by 4. The answer should come out as a decimal.</li> </ul>	<div style="display: flex; flex-wrap: wrap; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; margin: 5px; width: 45%;"> <p style="text-align: center;">Record the ideas for how they would split the money. (ex: exchanging the dollars for quarters and sharing them.)</p> <p style="text-align: right; font-size: small;">Chart It!</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 5px; width: 45%;"> <p style="text-align: center;">Record the ideas for how they would split the money using only dimes and pennies.</p> <p style="text-align: right; font-size: small;">Chart It!</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 5px; width: 45%;"> <p style="text-align: center;">Total share is \$243.75</p> <p style="text-align: right; font-size: small;">Chart It!</p> </div> <div style="border: 1px solid black; padding: 5px; margin: 5px; width: 45%;"> <p style="text-align: center;">Record the long division problem showing how to extend the remainder to a decimal.</p> <p style="text-align: right; font-size: small;">Chart It!</p> </div> </div> <p>At this point, you should have 3 ways to look at this problem that are posted side by side: long division with remainder, shortened version of repeated subtractions, and long division with decimals.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: 30%;"> <p style="text-align: center;">Record the answer, expressing the remainder as a fraction.</p> <p style="text-align: right; font-size: small;">Chart It!</p> </div>

**Activities**

Different Faces of Division (continued)	Notes
<ul style="list-style-type: none"> <li>• Next show them how to use the calculator to show a remainder instead.</li> <li>• The third time, show them how to display the answer with a fraction.</li> <li>• Ask them how the calculator gets the fraction.</li> <li>• Have a volunteer show where the fraction is on the long division problem.</li> <li>• Point out that these are all ways to write the quotient. Explain what a quotient is, and then ask participants what this quotient represents.</li> </ul> <p><b>Applications</b></p> <ol style="list-style-type: none"> <li>1. Give participants a chance to think about when they might want to know the whole number remainder, the fraction or the decimal.</li> <li>2. Hand out <b>Applications of Division</b>.             <ul style="list-style-type: none"> <li>• Have participants look at the problems and discuss what they all have in common.</li> <li>• What are the ways to represent the answer to this problem?</li> <li>• Tell them that they are going to read each situation and determine which form of the answer is most helpful in that situation.</li> </ul> </li> <li>3. Have participants share their reasoning for choosing each correct representation.</li> <li>4. If a few participants have done the challenge problem, have them share their problems and see if the class can decide which form of the quotient is more helpful for the problem.</li> </ol>	<div data-bbox="1133 279 1333 590" style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;">  <p style="text-align: center; color: green; font-weight: bold;">Quotient is the answer to a division problems. In this case, the quotient represents equal portions.</p> <p style="text-align: right; color: blue; font-size: small;">Chart It!</p> </div> <p style="margin-top: 20px;">They should notice that every problem is the same division problem.</p>
Mathematical Reasoning (15-20 minutes)	Notes
<p>Tell participants that since the focus of this session has been division, the mathematical reasoning will be about division.</p> <ol style="list-style-type: none"> <li>1. Hand out <b>Division Principles</b>.             <ul style="list-style-type: none"> <li>• Tell participants that they will be working with conjectures again. Conjectures are examined through examples and counter-examples. When working with conjectures, the object is to find a counter-example.</li> <li>• Have them look at problem 1 and ask if the following statement is true or false:</li> </ul> </li> </ol>	<p>If participants are stuck on the meaning of the statement, have someone show an example of numbers that fit the statement and talk through what it means.</p>

**Activities**

Mathematical Reasoning (continued)	Notes
<ul style="list-style-type: none"> <li>• If <math>A &gt; B</math> and <math>B &gt; C</math>, then <math>A &gt; C</math>.</li> <li>• Ask them to look for examples and counter-examples of the statement (Encourage them to use calculators).</li> <li>• After some minutes, have a volunteer explain their thinking about this statement, showing examples and/or counter-examples.</li> <li>• Ask the class to make sense of this statement using a model or a story problem.</li> <li>• Choose one story problem and one model for presentation.</li> </ul> <p>2. Have participants see if the statement is true if <math>B &lt; C</math> instead. The new statement would read; If <math>A &gt; B</math> and <math>B &lt; C</math>, then <math>A &gt; C</math>. Follow the same procedure for reporting out.</p> <p>3. Have the participants explore #2 on the sheet.</p> <ul style="list-style-type: none"> <li>• After several minutes, have volunteers report numbers that they have found and record them in a table on the overhead. Rearrange them in numerical order.</li> <li>• Ask participants to look at the list to find patterns.</li> <li>• See if they can identify any missing numbers.</li> <li>• See if they can guess other numbers that will be on the list.</li> <li>• Ask volunteers to share what they have discovered.</li> </ul>	<p>If desired, remind participants that <math>&lt;</math> means “less than” and <math>&gt;</math> means “greater than”.</p> <p>Participants should discover that the numbers get larger by 6 each time. The table should include 5, 11, 17, 23, 29, and 35 at least. They may need nudges to get to the idea that the numbers take the form of <math>6x - 1</math> (however, accept <math>6x + 5</math> as well). One way to do this is to rewrite the numbers with the 6 pattern evident: example: <math>5 = 6 - 1</math>, <math>11 = 6 + 6 - 1</math>, <math>17 = 6 + 6 + 6 - 1</math>, <math>23 = 6 + 6 + 6 + 6 - 1</math>, and so forth. Written in this manner, they can see the multiplication of 6. This can also be tied back into the last session on multiplication as repeated addition.</p>
<p><b>Closure (15-20 minutes)</b></p>	
<p>Participants reflect on the activities of this session and their experience with division.</p> <ul style="list-style-type: none"> <li>• Direct them to share with a partner one mathematical idea they learned or saw differently as a result of this session.</li> <li>• Circulate and listen to this sharing.</li> <li>• Ask a few volunteers to share their reflections.</li> <li>• Record significant insights.</li> </ul>	

**Activities**

Take Home Activities (5 minutes)	Notes
<p>1. There are six handouts for participants to take home:</p> <ul style="list-style-type: none"> <li>• <b>Bringing Mathematics Home 3</b></li> <li>• <b>Division with Whole Numbers</b></li> <li>• <b>Money Templates</b></li> <li>• <b>Sharing Money Equally</b></li> <li>• <b>Applications of Division</b></li> <li>• <b>Division Principles.</b></li> </ul> <p>2. Have participants look through the packet of materials as you explain them. One purpose of the take home activities is for them to practice division with their children. They have templates for play money and fresh copies of the session's activities for this purpose.</p> <p>3. There are also some new challenges for the participants in <b>Division with Whole Numbers</b>.</p> <p>4. Let participants know that they should be ready to share their experiences at the next session.</p>	
Preparation for the Next Session (5 minutes)	
<p>1. Collect name cards for use in the next sessions.</p> <p>2. Save the <b>Chart It!</b> and bring it to the next class. If desired, you may have the log typed and distributed to participants at the next class.</p>	

**Facilitator Notes**

**Recording Division: Answer Key to *Sharing Money Equally***

1st person	2nd person	3rd person	4th person	Money Used	Exchanges
\$100	\$100	\$100	\$100	\$400	10 \$10's for \$100
\$100	\$100	\$100	\$100	\$400	
\$10	\$10	\$10	\$10	\$40	10 \$1's for \$10
\$10	\$10	\$10	\$10	\$40	
\$10	\$10	\$10	\$10	\$40	
\$10	\$10	\$10	\$10	\$40	
\$1	\$1	\$1	\$1	\$4	\$3 left over
\$1	\$1	\$1	\$1	\$4	
\$1	\$1	\$1	\$1	\$4	
\$243	\$243	\$243	\$243		\$3

So each person gets \$243 and there are \$3 left over.

**Scaffolding**

Recording \$	Each Person Receives		
\$ 975		Record these	Then shorten it,
- 400	\$100	two methods	making connections
575		simultaneously	for each number.
- 400	100		
175		1	243
- 40	10	1	4 $\overline{)975}$
135		1	<u>-800</u>
- 40	10	10	175
95		10	<u>-160</u>
- 40	10	10	15
55		10	<u>- 12</u>
- 40	10	100	3
15		100	
- 4	1		
11		4 $\overline{)975}$	
- 4	1		
7			
- 4	<u>1</u>		
3	243		