SESSION TW0
INTRODUCTION TO ALGEBRAIC EXPRESSIONS

Outcomes

- To investigate geometrical patterns and relationships with models
- To use a table to record patterns and relationships
- To introduce variables and algebraic expressions
- To make predictions using a table or expression

Overview

This session builds on the experiences of the first session by asking participants to use their understanding of piles of cubes to begin writing informal rules. These rules are converted from words to algebraic expressions. Participants then apply their understanding of writng expressions to new situations.

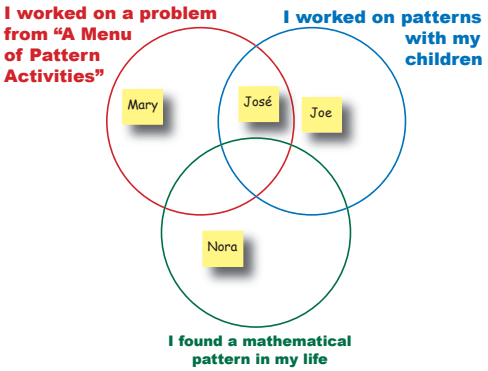
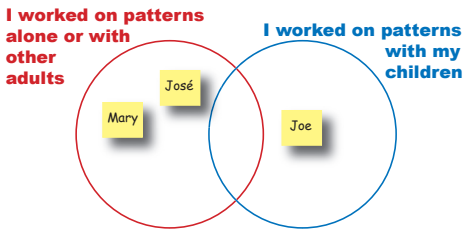
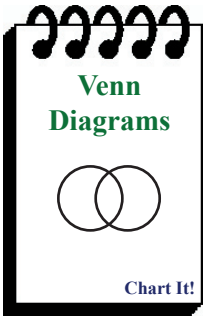
Time

10–15 minutes	As participants arrive they complete a Venn Diagram that shows the type of work they did at home relating to patterns. A discussion follows about the information that can be drawn from the diagram and about the use of diagrams in mathematics classrooms. Examples of tables from the homework assignment are discussed.
20–30 minutes	Next, participants briefly recall the mathematics from the first session. They revisit a problem similar to the **Piles of Cubes I** activity from the first session. They make general statements about how to find the number of cubes in a pile and use a variable to write an algebraic expression.
30-40 minutes	Participants are then given similar problems involving hexagons and triangles. They complete a table, investigate the patterns and determine an algebraic expression for each. They present their work to the class.
15–25 minutes	Participants revisit another problem from the first session, Flying Vs and Ws. They use their understanding of tables, patterns, and variables to write algebraic expressions. They present their work to the class.
5–10 minutes	The session ends with participants reflecting on the mathematics of the session. Assignments for home are given.

Materials

Facilitator	Transparencies (Eng. & Spanish)
• Poster size version of Venn Diagrams (see sample under Preparation of Classroom) • Post its for Venn Diagram activities • Poster size copies of **Piles and Cubes I** and Flying Vs and Ws in **A Menu of Pattern Activities** (from session 1)	*BLM 8: From Relationships to Expressions*
Participant	**Handouts (English & Spanish)**
• Pattern Blocks, 1 set per table • Colored cubes (any type), 1 set per table	**One per participant for class** *BLM 5: Piles of Cubes II* *BLM 6: Triangles* *BLM 7: Hexagons* *BLM 8: From Relationships to Expressions*

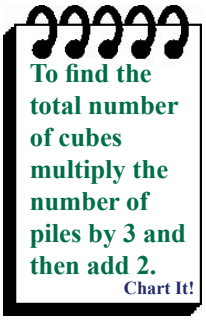
Activities

Preparation of Classroom	Notes
1. Set up the **Chart It!** from the first session. 2. If the names cards were collected in the first session, place them on the tables. 3. Use poster paper to create a Venn Diagram that asks parents to show the type of work they did from the **At Home with Patterns** assignment. Use diagrams similar to the ones to the right. Post them on the wall. Place post-its and markers next to the poster. 4. If classes are bilingual, write the Spanish translation on the top arcs of the circles and the English translation on the bottom arcs. In bilingual classes, it is best to alternate the languages that appear first or above the other.	 A simpler version of a Venn Diagram which can be used is shown below: 
Discussion of Homework (10-15 minutes)	
1. As parents arrive give them a small post-it, have them write their name on it and ask them to place their name in the appropriate place on the Venn diagram. Hand out transparencies to participants who have brought tables. 2. After all parents arrive lead a discussion about the Venn Diagrams. Some examples of questions that will get them to think about the information posted are: • *How many people worked with their children on patterns?* • *What do you know about the people whose names are in this section (pointing to a section)?* • *Are there any people in the room who did not...(list activities)?* If no names are outside the circle, ask: • *What would it mean if we had names outside of the circles?* 3. Let them know that these are called Venn diagrams. Write the term on **Chart It!** and draw a model of one.	Post-its are preferable to writing names directly on the chart paper. If errors occur in the posting they can be easily corrected. It is helpful to stand by the diagram and ask questions that will help the parents determine where to place their post-it. 

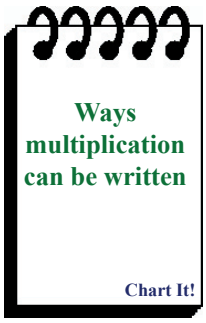
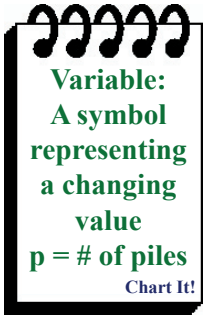
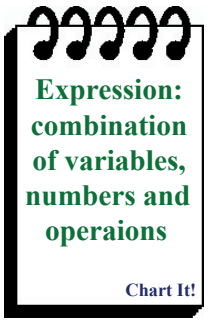
Activities

Discussion of Homework (continued)	Notes
4. This is an appropriate time to briefly discuss how and when Venn diagrams are used in the current curriculum of the participating schools. 5. Have participants share some of the tables that they created as part of their homework assignment. Many particpants get confused about how to label the left column, so spend some time looking at their examples. Discuss how one can decide on the labels of the left-hand column. 6. Take a few minutes to have the participants recall the mathematics that they did in the first session referring to the charts from the previous session. If needed, complete the presentations from the first session. Decide on the amount of time to spend on this based on participants' comments and questions. 7. Let participants know that they will use their understanding of patterns and tables to generalize relationships into rules and make predictions.	
Piles of Cubes II (15-20 minutes)	
1. Distribute the **Piles of Cubes II** handout. Remind participants that they did a similar problem at the first session. Have cubes available on the tables for their use. 2. Have participants complete questions 1-3. 3. Ask: • *How many cubes are in the 10th pile?* • *How did you figure this out?* 4. Record the responses in the manner in which the participants respond. (e.g., "I multiplied 10 by 3 and added 2" = 3 x 10 +2; "I added 2 to 3 times 10 "+ 2 + 3 x 10.)	The purpose of this activity is to write a rule for finding the number of cubes in a pile. It is helpful to generalize the rule in words before variables are introduced. Some participants may not need to build the piles in order to complete the table. It is not necessary that they do so. Those that need to build will build. Some strategies that are used are building, drawing, completing the table, and imagining the pile. Their answers will be similar to • I completed the table to find the 10th pile. • I drew a picture of it. • I know that there are ten 3s plus two more.

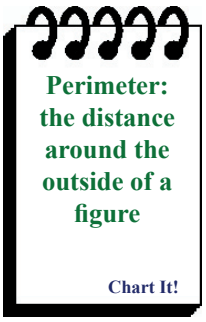
Activities

Piles and Cubes Revisited (continued)	Notes
5. Ask: 20th pile: • *How many cubes are in the 20th pile?* • *How did you figure this out?* 100th pile: • *If we were to build the 100th pile, what would it look like?* • *How many cubes are in the 100th pile?* • *How did you figure this out?* • *Is there another way other than adding 3 to the previous pile to find the number of cubes in the 100th pile since you would need to know the number of cubes in the 99th pile to do this?* Record the responses to these questions on **Chart It!**. 6. Ask; *If we were to build the 1000th pile, what would you do to find the number of cubes?* When they express that you just need to multiply the number of piles by 3 and then add 2, ask: *So, no matter how many piles you have what do you need to do to find the total number of cubes?* 7. Record their response in words similar to the following: *"To find the total number of cubes multiply the number of piles by 3 and then add 2."* 8. Let them know that we have a way to write this relationship between the number of piles and cubes. Say: *Let's use p to represent the number of piles. If p is the number of piles, what do we need to do to p to get the total number of cubes? How can you write this?* Have participants work in their groups to find a way to write an expression. 9. Ask participants to share their ideas. Ask if anyone wrote it differently. Have them write their expressions on chart paper.	Many of the participants will only know the use of an "x" to show multiplication. Use the "x" now. The discussion on other methods to write multiplication will come up later.  Some participants see the pattern as three times the number of piles plus 1, then subtract 1 or 3(p+1) – 1.  Using a variable that corresponds to the context of the problem makes it easier for the participants to grasp the meaning of a variable. It is best to get the participants to share their ways instead of telling them all of the ways. If all the ways are not shared, then share them with the participants. Expressions given by parents are usually: 3 x p + 2 p x 3 + 2 2 + 3 x p Parents rarely write the expression 3p+2.

Activities

Piles and Cubes Revisited (continued)	Notes
10. Use this opportunity to discuss the various ways multiplication can be written: • The use of parenthesis to show which operation to do first, e.g., (3 x p) + 2 • The use of parenthesis to show multiplication, e.g., (3) (p) + 2 • The use of a dot or asterisk to show multiplication, e.g., 3 • p or 3∗ p • The use of a constant with a variable, e.g., 3 p , to show multiplication	The discussion on many ways to show multiplication has always been of interest to the participants. It is a topic that needs to be revisited throughout this class. 
11. Once an expression is determined, record it and have them work in groups to use the expression to verify that it works for the data they have in their tables.	
12. Have them predict the number of cubes for 16 piles using the expression. They can verify that they are correct by building, drawing, extending their table to 16 piles, etc.	
13. Let participants know that when we use letters to represent numbers those letters are called variables. Add the term variable to **Chart It!**. Say: *They are called variables because the value of the number changes or varies.*	The term, variable, is intentionally introduced after the participants develop an understanding of the concept.
14. In order to help participants make connections between the model and the rule, ask: • *Your rule is _____ (e.g. p times 3 plus 2).* • *Using the model can you tell why there is a 2? A 3?* • *Where do these numbers show up in the model?*	 
15. Ask: • *If the model had only one cube at the top instead of 2, would the rule change? How?* • *If the model had 6 cubes across instead of 3, how would the rule change?*	
16. Write "5 times a number, p" in words. Ask the participants to talk to a partner about how they could write this statement mathematically. After giving them a minute to talk about it, ask for suggestions. List their ideas and add any that may be missing. Also, explain the term expression and record it on the list. Add an example for each.	

Activities

Triangles and Hexagons (30-40 minutes)	Notes
1. Distribute the handouts, **Triangles** and **Hexagons**. Have them use the pattern blocks to make the patterns, complete the table and write an algebraic equation. 2. Make sure that the participants understand that the triangles and/or hexagons are to be laid side-by-side and form a line. This is an important discussion as some participants want to connect the triangles using a different pattern. 3. Have a brief discussion about perimeter. Record the word and definition on **Chart It!**. Let participants know that the length of one side of a triangle will be used to measure the perimeter. Thus, a triangle has a perimeter of 3 units. 4. Let the participants work in pairs or groups to find an expression for each of the problems. 5. As you work with groups on these problems, you may notice that different groups have developed various equivalent expressions. Decide on the order of group presentations based on the simplicity of the rule, calling on the simplest rule first. 6. Have the groups present their work. Leading applause after a presentation is appropriate. 7. After each presentation of a rule for the pattern, ask the class if they agree that this expression works for the pattern. Have someone prove that it works. 8. After the class has presented all their expressions ask: *If these all work then are they all equal to each other?* You will need to decide about the depth of discussion that you will lead based on the mathematical experiences of the parents. You may want to state that more in-depth algebraic experiences would be needed to prove that they are all equivalent.	 If some finish before others, challenge them to make their own pattern with the other pattern blocks and find a rule for the pattern. A particular challenging problem is to find the perimeter of the **Piles of Cubes** (multiply the pile number plus 1 by 2 then add 6). Examples of rules: **t + 2***:* Some determine this rule by looking at the table and determining that the number of triangles plus 2 give the perimeter. Others determine this rule by counting the tops and bottoms and adding the two sides. **3 +1(t-1):** Begin with 3 sides and add 1 each time you add a triangle. The number of 1s added is always 1 less than the number of triangles. **3t – 2(t-1):** Each triangle has 3 sides so multiply t by 3. When you put triangles together you lose 2 sides. The number of 2s you lose is always one less than the number of triangles. The rules and explanations for the hexagons are similar.

Activities

Flying Vs and Ws (15-25 minutes)

1. If time permits, ask participants to refer to the Flying Vs and Ws problems from the first session and have them write an algebraic expression for both.

2. As with the previous problems there will be various equivalent expressions for each. Some examples are:

Flying Vs	Flying Ws
1 + 2n 3 + 2(n - 1) (n + 1) + n	5 + (n - 1)4 2(1 + 2n) - 1

3. Have groups present their work using chart paper or transparencies. Ask the following types of questions to enhance the discourse in the class.

- *Where did you get that 3 in the expression?*
- *Why did you multiply by 2?*
- *Are those two expressions equivalent?*
- *If you know how to find the number of birds, can that help you find the number of airplanes?*

Notes

It is not necessary that all participants work both problems. When the time is appropriate ask for volunteers to share their thinking.

Some possible explanations are:

Flying Vs

1 + 2n:

The 1 is the vertex of the v, and the two sides of the v have the same number of birds.

3 - 2(n - 1):

Each has a set of 3 at the base of the v, left are sets of 2 (the number of sets of 2 is one less than the formation number.

(n + 1) + n:

1st - 2 on one side plus 1

2nd - 3 on one side plus 2

3rd - 4 on one side plus 3

Flying Ws

5 + (n - 1)4:

Each has a base of 5 and sets of 4 birds above the base; the number of 4s is one less than the formation number.

2(1 + 2n) - 1:

If the flying Vs has 1 +2n birds, then the Ws has 2 sets of Vs minus the one which would be counted twice.

Closure (5 minutes)

1. Ask participants to think about the steps that they did to find an expression. Record their thinking.

2. Display the transparency of **From Relationships to Expressions** and distribute handout to participants. Use this to briefly summarize the experiences of the past two sessions.

3. Ask the participants to review the terms on **Chart It!** and choose one term. Have them explain their understanding about that term to a partner.

Activities

Take Home Activities (5 minutes)	Notes
Ask participants to revisit Toothpick Houses in ***A Menu of Activities*** from session one. If there was not sufficient time to write algebraic expressions for the Flying Vs and Ws problems, you may ask them to write expressions for both Toothpick Houses and the Flying Vs and Ws problems.	
Preparation for the Next Session	
1. Collect name cards if applicable. 2. Save the **Chart It!** and bring it to the next class. 3. Optional: • Type the notes on the **Chart It!** and distribute at the next session. • Take digital pictures of each chart. Prepare handouts of these pictures.	