

# Session One: Introduction to Fractions, Decimals, and Percents

## Common Core Standards Addressed

### Grade 3

#### Develop understanding of fractions as numbers.

- **CCSS.Math.Content.3.NF.A.1** Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ .
- **CCSS.Math.Content.3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
  - **CCSS.Math.Content.3.NF.A.3a** Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
  - **CCSS.Math.Content.3.NF.A.3b** Recognize and generate simple equivalent fractions, e.g.,  $1/2 = 2/4$ ,  $4/6 = 2/3$ . Explain why the fractions are equivalent, e.g., by using a visual fraction model.
  - **CCSS.Math.Content.3.NF.A.3c** Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form  $3 = 3/1$ ; recognize that  $6/1 = 6$ ; locate  $4/4$  and 1 at the same point of a number line diagram.*
  - **CCSS.Math.Content.3.NF.A.3d** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

#### Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

- **CCSS.Math.Content.3.MD.C.5** Recognize area as an attribute of plane figures and understand concepts of area measurement.
  - **CCSS.Math.Content.3.MD.C.5a** A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
  - **CCSS.Math.Content.3.MD.C.5b** A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units.
- **CCSS.Math.Content.3.MD.C.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
- **CCSS.Math.Content.3.MD.C.7** Relate area to the operations of multiplication and addition.
  - **CCSS.Math.Content.3.MD.C.7a** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
  - **CCSS.Math.Content.3.MD.C.7b** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
  - **CCSS.Math.Content.3.MD.C.7c** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths  $a$  and  $b + c$  is the sum of  $a \times b$  and  $a \times c$ . Use area models to represent the distributive property in mathematical reasoning.

- **CCSS.Math.Content.3.MD.C.7d** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

### Reason with shapes and their attributes.

- **CCSS.Math.Content.3.G.A.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- **CCSS.Math.Content.3.G.A.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as  $1/4$  of the area of the shape.*

## Grade 4

### Extend understanding of fraction equivalence and ordering.

- **CCSS.Math.Content.4.NF.A.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
- **CCSS.Math.Content.4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### Build fractions from unit fractions.

- **CCSS.Math.Content.4.NF.B.3** Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .
  - **CCSS.Math.Content.4.NF.B.3a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
  - **CCSS.Math.Content.4.NF.B.3b** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:  $3/8 = 1/8 + 1/8 + 1/8$ ;  $3/8 = 1/8 + 2/8$ ;  $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$ .*
  - **CCSS.Math.Content.4.NF.B.3c** Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
  - **CCSS.Math.Content.4.NF.B.3d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
- **CCSS.Math.Content.4.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
  - **CCSS.Math.Content.4.NF.B.4a** Understand a fraction  $a/b$  as a multiple of  $1/b$ . *For example, use a visual fraction model to represent  $5/4$  as the product  $5 \times (1/4)$ , recording the conclusion by the equation  $5/4 = 5 \times (1/4)$ .*

- **CCSS.Math.Content.4.NF.B.4b** Understand a multiple of  $a/b$  as a multiple of  $1/b$ , and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express  $3 \times (2/5)$  as  $6 \times (1/5)$ , recognizing this product as  $6/5$ . (In general,  $n \times (a/b) = (n \times a)/b$ .)*
- **CCSS.Math.Content.4.NF.B.4c** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat  $3/8$  of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

## Grade 5

### Use equivalent fractions as a strategy to add and subtract fractions.

- **CCSS.Math.Content.5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,  $2/3 + 5/4 = 8/12 + 15/12 = 23/12$ . (In general,  $a/b + c/d = (ad + bc)/bd$ .)*
- **CCSS.Math.Content.5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result  $2/5 + 1/2 = 3/7$ , by observing that  $3/7 < 1/2$ .*

### Apply and extend previous understandings of multiplication and division.

- **CCSS.Math.Content.5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret  $3/4$  as the result of dividing 3 by 4, noting that  $3/4$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $3/4$ . If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
- **CCSS.Math.Content.5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
  - **CCSS.Math.Content.5.NF.B.4a** Interpret the product  $(a/b) \times q$  as a parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . *For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)*
  - **CCSS.Math.Content.5.NF.B.4b** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
- **CCSS.Math.Content.5.NF.B.5** Interpret multiplication as scaling (resizing), by:
  - **CCSS.Math.Content.5.NF.B.5a** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
  - **CCSS.Math.Content.5.NF.B.5b** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole

numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence  $a/b = (n \times a)/(n \times b)$  to the effect of multiplying  $a/b$  by 1.

- **CCSS.Math.Content.5.NF.B.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- **CCSS.Math.Content.5.NF.B.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.<sup>1</sup>
  - **CCSS.Math.Content.5.NF.B.7a** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for  $(1/3) \div 4$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $(1/3) \div 4 = 1/12$  because  $(1/12) \times 4 = 1/3$ .*
  - **CCSS.Math.Content.5.NF.B.7b** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for  $4 \div (1/5)$ , and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that  $4 \div (1/5) = 20$  because  $20 \times (1/5) = 4$ .*
  - **CCSS.Math.Content.5.NF.B.7c** Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $1/3$ -cup servings are in 2 cups of raisins?*

## Grade 6

### Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

- **CCSS.Math.Content.6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi?.*

### Solve real-world and mathematical problems involving area, surface area, and volume.

- **CCSS.Math.Content.6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems